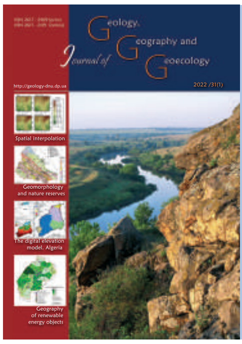

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Application of the Data Envelopment Analysis method to assess the efficiency of social and economic systems

Anna O. Kniazevych [1], Alexandr V. Kraychuk[2], Serhii O. Kraychuk [2], Nataliia O.Ostapchuk[2]

[1] *Stepan Demianchuk International University of Economics and Humanities, 4 Stepana Demianchuka street, Rivne, 33027, Ukraine,* kniazevich.a@gmail.com
[2] *Rivne State University of Humanities, 12 Bandery street, Rivne, 33028, Ukraine,* a.kraychuk@gmail.com, laxxik@gmail.com, ostanatata@gmail.com

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Abstract. The article deals with the relevant scientific and practical task – assessing the effectiveness of various types of existing systems by methods of mathematical modeling. In particular, the research investigates the efficiency of functioning of social and economic systems using nonparametric methods of mathematical modeling. For the verification of the results, the research uses the Data Envelopment Analysis (DEA), which is a modern method based on the creation of the efficiency limit. The article considers in detail the theoretical foundations of the DEA method, which is based on the problems of mathematical programming, namely, the problems of linear programming. The article presents main theoretical models of this nonparametric method for input and output and analyzes the technique of its application. As an example of the practical use of this method, the research suggests the analysis of the demographic situation in the regions of Ukraine based on the main demographic indicators – mortality and birth rate for a certain estimated period. In the process of analyzing the results of modeling, the research has identified two groups of regions – one with the most effective indicators of the demographic situation, and the second – with the least effective indicators. The selected groups of regions are invariant when using different models of the method. The calculations obtained in the process of modeling confirm the real condition of the demographic situation in the regions of Ukraine, which is characterized by the actual statistical indicators of the population in regions. For each region, the research has established reference regions, which in their structure are similar to the structure of indicators of given regions and are a certain model and reference point for the practical improvement of the demographic situation. The recommended values of indicators that set the direction of improving the demographic situation in each specific region of Ukraine have also been calculated during the research.

Keywords: social and economic systems, Data Envelopment Analysis method, nonparametric methods of mathematical modeling, efficiency limit, region, demographic indicators.

Застосування методу Data Envelopment Analysis до аналізу соціально-економічних систем

А.О.Князевич[1], О.В. Крайчук[2], С.О. Крайчук[2], Н.О. Остапчук[2]

[1] *ПВНЗ «Міжнародний економіко-гуманітарний університет імені академіка Степана Дем'янчука», Рівне, Україна,* kniazevich.a@gmail.com
[2] *Рівненський державний гуманітарний університет, Рівне, Україна,* a.kraychuk@gmail.com, laxxik@gmail.com, ostanatata@gmail.com

Анотація. Робота присвячена вивченню актуальної науково-практичної задачі – оцінки ефективності функціонування різного виду існуючих систем методами математичного моделювання. Досліджується, зокрема, ефективність функціонування соціально-економічних систем за допомогою непараметричних методів математичного моделювання. Для перевірки застосовано сучасний метод, що ґрунтується на побудові границі ефективності – метод Data Envelopment Analysis (DEA). У статті детально розглянуто теоретичні основи методу DEA, що ґрунтується на задачах математичного програмування, а саме, задачах лінійного програмування. Наведено основні теоретичні моделі даного непараметричного методу на вхід і вихід та проаналізовано методику його застосування. У якості практичного використання вказаного методу проведено вивчення стану демографічної ситуації в областях України на основі основних демографічних показників – смертності та народжуваності за певний розрахунковий період. У процесі аналізу результатів моделювання виділено дві групи областей – одну із найбільш ефективними показниками демографічної ситуації, а другу – з найменш ефективними. Виділені групи областей є інваріантними при застосуванні різних моделей методу. Одержані в процесі моделювання розрахунки підтверджують реальний стан демографічної ситуації в областях України, який характеризується фактичними статистичними показниками народонаселення в регіонах. Для кожної області встановлені еталонні області, які за своєю структурою подібні до структури показників заданих областей та є певним зразком і орієнтиром для практичного поліпшення демографічної ситуації. Обчислені також рекомендовані значення показників, що задають напрями поліпшення демографічної ситуації у кожній конкретній області України.

Ключові слова: соціально-економічні системи, метод Data Envelopment Analysis, непараметричні методи математичного моделювання, границя ефективності, регіон, демографічні показники.

Introduction

One of economic activity drivers, which determines the volume of production of goods and services designed to meet the needs, is labor resources. These are capable of working population of the country that has the physical and spiritual abilities necessary to participate in employment. The number and structure of labor resources are important. Their quality depends on the gender, professional qualification and cultural-educational levels. Therefore, in order to assess, for example, the number of labor resources, there is a need to analyze the social and demographic situation in Ukraine.

Formation of the title, authors and affiliations. Presently, one of the urgent problems is to assess the effectiveness of various social and economic systems. To solve this problem, different approaches, parametric and non-parametric modeling techniques are used. This research is focuses on the applied aspect of the method based on the construction of the efficiency limit (Coelli, Prasada Rao, Battese, 1998; Cooper, Seiford, Tone, 2000). In this case, those objects that produce the maximal number of expected results (outputs) with a given number of available prerequisites (inputs) are considered effective, and the points of inputs and outputs that correspond to them in space are on the verge of efficiency. If the point corresponding to the object under the study does not belong to the efficiency limit, then such objects function inefficiently and their activity needs to be adjusted. The inefficiency of the object under the study is directly proportional to the distance of the corresponding point from the efficiency limit. There is no obvious limit to effectiveness, so in practice it needs to be assessed in some way. One of the methods that allows making such an assessment is the method of Data Envelopment Analysis.

The analysis of the recent research and publications

Data Envelopment Analysis (DEA) is a non-parametric method based on linear programming problems. It was developed in 1978 by American scientists A. Charnes, W. W. Cooper, E. Rhodes (Charnes, Cooper, Rhodes, 1978), using the ideas of M. J. Farrell on assessing the efficiency of a unit of final product with one input and one output indicator (Farrell, 1957).

There is still no single definition for the DEA method in the Ukrainian scientific literature, although much attention has been paid to it recently. In the works (Khailuk, 2010), it is defined as "Envelopment Surface Method" or "Data Convolution Method", in (Otenko, 2013) – as "Operating Environment Analysis or Data Shell Method", in (Andriichuk, 2005; Andriichuk, 2011; Bilych, 2014) – as "Data Shell Analysis Method". In this paper, the abbreviation "DEA method" is used.

The purpose of the article is to study theoretically the models of the DEA method for assessing the effectiveness of social and economic systems and practical comparative analysis of the demographic situation in different regions of Ukraine.

Results and discussion

I. Basic models of the DEA method

The DEA method is used to assess the effectiveness of homogeneous objects (organizations, enterprises, regions, sectors of the national economy) in different social and economic systems. Let us consider in detail the algorithm for constructing a mathematical model of the DEA method.

Let us select K of homogeneous objects that are characterized by two types of parameters x_i – input (i=1,2,…, n) and y_j – output (j=1,2,…, m). Column vectors are specified for each of the k objects with coordinates x_i і y_j correspondently. Then the matrix X of dimension $n \times k$ is the matrix of input parameters for all k objects, and matrix Y of dimension $m \times k$ is the matrix of output parameters for these objects. This is the task for mathematical programming, which, applying the theory of duality, can be formulated as follows (Coelli, Prasada Rao, Battese, 1998).

To find $min_{\theta,\lambda}(\theta)$ for given constraints

$$\begin{cases} -y_j + Y\lambda \geq 0 \\ \theta x_i - X\lambda \geq 0 \\ \lambda \geq 0 \end{cases} \quad (1)$$

where θ – scalar, and λ is a vector of dimensional constants $k \times 1$. The value of θ obtained in the process of solving the problem is a measure of the efficiency of k^{th} (k=1,2,…, K) object and therefore does not exceed one, i. e. $\theta \leq 1$. This task is solved K times for each of the selected objects. Objects for which $\theta = 1$ are on the efficiency boundary, which is a convex polygon. Points corresponding to the objects, for which $\theta < 1$, can be projected to the efficiency limit. In this case, each of these points will be a linear combination of vectors $X\lambda$ and $Y\lambda$. Some coordinates of the vector λ have zero values. These coordinates correspond to those objects that are the reference for the evaluated object. A linear combination of reference objects forms an imaginary object, the coordinates of which determine the goals for a real inefficient object, indicating ways to improve its functioning.

Therefore, due to the application of the DEA method, for objects with $\theta < 1$, it is possible to formulate a way to increase the efficiency of these objects by proportional reduction of the input costs by the value of θ while maintaining the original results. The closer the point corresponding to the object under the study is to the established efficiency limit, the higher the efficiency of this object is (Coelli, Prasada Rao, Battese, 1998).

Formula (1) is an input-oriented model with a constant scale effect (CRS – model). In order to take into account the possibility of changing the scale of the effect, namely, to form a convex linear combination of reference objects, it is necessary to add a condition $\sum_{k=1}^{K}\lambda_k = 1$ to this model – constraint of the convexity of a polyhedron of solutions of a linear programming problem. This is the task of mathematical programming (Coelli, Prasada Rao, Battese, 1998):

To find $min_{\theta,\lambda}(\theta)$ for given constraints

$$\begin{cases} -y_j + Y\lambda \geq 0 \\ \theta x_i - X\lambda \geq 0 \\ \sum_{k=1}^{K}\lambda_k = 1 \\ \lambda \geq 0 \end{cases} \quad (2)$$

Model (2) is an input-oriented model with variable scale effect (VRS – model).

Models (1) and (2) are input-oriented models. It is also possible to build output-oriented mathematical models, the task of which will be to increase the output without increasing the input costs (Coelli, Prasada Rao, Battese, 1998):

To find $max_{\phi,\lambda}(\phi)$ for given constraints

$$\begin{cases} -\phi y_j + Y\lambda \geq 0 \\ x_i - X\lambda \geq 0 \\ \lambda \geq 0 \end{cases} \quad (3)$$

To find $max_{\phi,\lambda}(\phi)$ for given constraints

$$\begin{cases} -\phi y_j + Y\lambda \geq 0 \\ x_i - X\lambda \geq 0 \\ \sum_{k=1}^{K}\lambda_k = 1 \\ \lambda \geq 0 \end{cases} \quad (4)$$

In the process of calculations, models (3) and (4) for each object determine the values of the efficiency factor and the values of the initial parameters, at which inefficient objects are brought to the efficiency limit, i.e. begin to function effectively. Model (3) is a model focused on the output with a constant scale effect (CRS – model), and model (4) is a model focused on the output with a variable scale effect (VRS – model). In these models, the variable ϕ takes values greater than or equal to one, i.e. $\phi \geq 1$. To calculate the traditional value of the efficiency indicator, the value $1/\phi$ inverted to ϕ is used (Coelli, Prasada Rao, Battese, 1998). It belongs to the interval [0.1], i.e. varies from zero to one.

II. Modeling of the demographic situation in the regions of Ukraine

To demonstrate the application of the DEA method, we will assess the demographic situation in the regions of Ukraine based on the basic demographic indicators – mortality and birth rate for a certain period (Table 1).

We use *x* and *y* as input and output indicators, respectively:

x – number of registered deaths;

y – number of registered live births.

Table 1. The initial data for calculation – The main demographic indicators of the regions of Ukraine, 2008–2018

No	Name of the region	2008		2013		2018	
		Number of registered deaths, *x*	Number of registered life births, *y*	Number of registered deaths, *x*	Number of registered life births, *y*	Number of registered deaths, *x*	Number of registered life births, *y*
1	Vinnytsia	28492	17915	25453	17437	24341	12769
2	Volyn	15594	15301	13666	14700	13710	11270
3	Dnipropetrovska	59781	37383	51141	36134	52336	25121
4	Donetsk	81948	44394	69345	41034	40174	15894
5	Zhytomyr	23760	14641	20859	15001	20227	10612
6	Zakarpatska	16155	18292	14801	18491	15320	13883
7	Zaporizhzhia	30127	18901	26498	18134	27871	12708
8	Ivano-Frankivsk	18385	16983	17358	16716	17449	12645
9	Kyiv	30946	20195	27198	20511	28722	15236
10	Kirovohradska	19272	10538	16513	10562	15484	7077
11	Luhansk	42181	22259	35822	20531	15991	5652
12	Lviv	35126	29007	31666	29542	32726	23253
13	Mykolaiv	19955	13378	17353	13043	17156	9141
14	Odesa	37951	28780	33523	29075	33607	23144
15	Poltava	27968	14748	24358	14296	23659	10193
16	Rivne	16245	17089	14556	17445	14528	13380
17	Sumy	22301	10835	19219	10411	17877	7114
18	Ternopil	16200	12388	14682	11807	15013	8545
19	Kharkiv	45109	27207	39465	26700	42600	19657
20	Kherson	18016	12473	16048	12300	16163	9095

Continuation of Table 1

No	Name of the region	2008		2013		2018	
		Number of registered deaths, *x*	Number of registered life births, *y*	Number of registered deaths, *x*	Number of registered life births, *y*	Number of registered deaths, *x*	Number of registered life births, *y*
21	Khmelnytskyi	22943	14822	20581	14548	19736	10698
22	Cherkasy	23386	12466	20477	12100	20181	8637
23	Chernivtsi	12194	11067	11520	11465	11259	8710
24	Chernihiv	23782	10039	19909	9852	19304	6854

Source: developed by the authors on the basis of statistical data of the Department of Statistics in Ukraine.

There is no direct relationship between these indicators, i.e. it is not possible to derive one quantitative value of one indicator from the quantitative value of another indicator and vice versa. For the input parameter *x*, obviously, a tendency to decrease is desirable, and for *y* – a tendensy to increase is desirable. Therefore, to study the interaction of indicators x and y it is possible to use the DEA method, which allows comparing the efficiency of transformation of the selected parameters.

To choose a way to increase population growth, we use DEA models that are focused on input, which will provide specific quantitative recommendations for reducing mortality – the model (1). Population growth is possible both by increasing the birth rate and by reducing mortality. However, demographic situations are possible in which high or low mortality and birth rates occur at the same time. These types of demographic situations are typical for African countries (high rates) and developed European countries (low rates). It can be assumed that there are also areas in Ukraine where demographic processes are similar. To compare regions and select reference regions, it is necessary to take into account the relevant types of demographic situations, which allows performing the model (2), which takes into account the variable scale of the effect.

Models (1) and (2) are used for modeling, and the modeling process itself is implemented with Excel software (Table 2).

Table 2. Indicators of the effectiveness of the demographic situations in the studied regions according to the INPUT models, 2008–2018

No	Name of the region	2008				2013				2018			
		crste	vrste	scale		crste	vrste	scale		crste	vrste	scale	
1	Vinnytsia	0.555	0.560	0.992	irs	0.548	0.562	0.975	irs	0.570	0.579	0.983	irs
2	Volyn	0.867	0.931	0.931	irs	0.861	0.954	0.903	irs	0.893	0.952	0.938	irs
3	Dnipropetrovska	0.552	1.000	0.552	drs	0.566	1.000	0.566	drs	0.521	1.000	0.521	drs
4	Donetsk	0.478	1.000	0.478	drs	0.474	1.000	0.474	drs	0.430	0.474	0.906	drs
5	Zhytomyr	0.544	0.596	0.914	irs	0.576	0.631	0.912	irs	0.570	0.622	0.915	irs
6	Zakarpatska	1.000	1.000	1.000	-	1.000	1.000	1.000	-	0.984	1.000	0.984	drs
7	Zaporizhzhia	0.554	0.572	0.969	drs	0.548	0.552	0.992	irs	0.495	0.504	0.982	irs
8	Ivano-Frankivsk	0.816	0.840	0.972	irs	0.771	0.805	0.958	irs	0.787	0.803	0.980	irs
9	Kyiv	0.576	0.631	0.914	drs	0.604	0.658	0.918	drs	0.576	0.621	0.928	drs
10	Kirovohradska	0.483	0.633	0.763	irs	0.512	0.698	0.734	irs	0.496	0.727	0.682	irs
11	Luhansk	0.466	0.550	0.848	drs	0.459	0.500	0.917	drs	0.384	0.704	0.545	irs
12	Lviv	0.729	1.000	0.729	drs	0.747	1.000	0.747	drs	0.771	1.000	0.771	drs
13	Mykolaiv	0.592	0.675	0.878	irs	0.602	0.706	0.852	irs	0.579	0.674	0.859	irs
14	Odesa	0.670	0.915	0.732	drs	0.694	0.923	0.752	drs	0.748	0.968	0.773	drs
15	Poltava	0.466	0.508	0.916	irs	0.470	0.527	0.891	irs	0.468	0.520	0.900	irs
16	Rivne	0.929	0.954	0.974	irs	0.959	0.983	0.976	irs	1.000	1.000	1.000	-
17	Sumy	0.429	0.547	0.785	irs	0.434	0.599	0.723	irs	0.432	0.630	0.686	irs
18	Ternopil	0.675	0.797	0.847	irs	0.644	0.796	0.809	irs	0.618	0.750	0.824	irs
19	Kharkiv	0.533	0.708	0.752	drs	0.542	0.692	0.782	drs	0.501	0.611	0.819	drs
20	Kherson	0.611	0.720	0.850	irs	0.614	0.742	0.827	irs	0.611	0.713	0.857	irs
21	Khmelnytskyi	0.571	0.621	0.918	irs	0.566	0.630	0.899	irs	0.589	0.641	0.918	irs
22	Cherkasy	0.471	0.554	0.849	irs	0.473	0.577	0.820	irs	0.465	0.558	0.833	irs
23	Chernivtsi	0.802	1.000	0.802	irs	0.797	1.000	0.797	irs	0.840	1.000	0.840	irs
24	Chernihiv	0.373	0.513	0.727	irs	0.396	0.579	0.685	irs	0.386	0.583	0.661	irs

Legend: crste – technical efficiency according to the model CRS DEA, vrste – technical efficiency according to the model VRS DEA, scale = crste / vrste, drs – decrease in efficiency from scale, irs – increase in efficiency from scale.

Source: authors' own research.

The efficiency indicators calculated using the VRS – model (2) show that the group of regions with a favorable demographic situation in 2018 includes: Zakarpatska, Rivne, Volyn, Chernivtsi regions. At the same time, Chernihiv, Sumy, Luhansk, Poltava regions had the lowest levels of demographic indicators. The efficiency indicators calculated using the VRS – model (2) show that the group of regions with a favorable demographic situation in 2013 includes: Zakarpatska, Rivne, Volyn, Chernivtsi regions. At the same time, Chernihiv, Sumy, Luhansk regions had the lowest levels of demographic indicators. The efficiency indicators calculated using the VRS – model (2) show that the group of regions with a favorable demographic situation in 2018 includes: Rivne, Chernivtsi, Zakarpatska, Lviv and Dnipropetrovska regions. At the same time, Luhansk, Chernihiv and Donetsk regions had the lowest levels of demographic indicators. These results are obtained when the input parameter is the number of registered deaths.

Let us now use output-oriented DEA models (Table 3), namely, the CRS model – (3) and the VRS model – (4). The obtained results are compared with the previous ones, calculated according to the input models.

Table 3. Indicators of the effectiveness of the demographic situations in the studied regions according to the OUTPUT models, 2018

No	Name of the region	2018			
		crste	vrste	scale	
1	Vinnytsia	0.674	0.708	0.952	drs
2	Volyn	0.430	0.502	0.857	irs
3	Dnipropetrovska	0.736	1.000	0.736	drs
4	Donetsk	0.893	1.000	0.893	drs
5	Zhytomyr	0.674	0.684	0.986	drs
6	Zakarpatska	0.390	0.407	0.958	irs
7	Zaporizhzhia	0.775	0.831	0.932	drs
8	Ivano-Frankivsk	0.488	0.489	0.998	drs
9	Kyiv	0.666	0.718	0.929	drs
10	Kirovohradska	0.773	0.799	0.968	irs
11	Luhansk	1.000	1.000	1.000	-
12	Lviv	0.497	0.545	0.913	drs
13	Mykolaiv	0.663	0.665	0.998	drs
14	Odesa	0.513	0.564	0.910	drs
15	Poltava	0.820	0.857	0.957	drs
16	Rivne	0.384	0.422	0.909	irs
17	Sumy	0.888	0.891	0.997	drs
18	Ternopil	0.621	0.661	0.939	irs
19	Kharkiv	0.766	0.902	0.849	drs
20	Kherson	0.628	0.628	1.000	-
21	Khmelnytskyi	0.652	0.658	0.991	drs
22	Cherkasy	0.826	0.838	0.986	drs
23	Chernivtsi	0.457	0.649	0.704	irs
24	Chernihiv	0.995	1.000	0.995	drs

Source: authors' own research.

Analyzing the results of modeling, we can distinguish two groups of regions. Luhansk, Donetsk, Chernihiv and Dnipropetrovska regions were the most efficient, while Zakarpatska, Rivne, Ivano-Frankivsk and Volyn regions were the least efficient. These results are obtained when the input parameter is the number of registered live births.

III. Analysis of the results of modeling

Having analyzed the results obtained with the help of the DEA input and output models, it can be concluded that the most favorable demographic situation is in Rivne and Zakarpatska regions, and the most unfavorable – in Luhansk, Donetsk and Chernihiv regions.

The results of the assessment of the effectiveness of the demographic situation in the regions of Ukraine are presented in the table 4.

Table 4. Efficiency assessment results and reference regions

No	Name of the region	Efficiency indicator	Place (ranking)	Reference regions and coefficients which help them to form a hypothetical object			
1	Vinnytsia	0.579	20	Rivne	0.869	Chernivtsi	0.131
2	Volyn	0.952	7	Rivne	0.548	Chernivtsi	0.452
3	Dnipropetrovsk	1.000	1	Dnipropetrovsk	1.000		
4	Donetsk	0.474	24	Zakarpatska	0.785	Lviv	0.215
5	Zhytomyr	0.622	16	Rivne	0.407	Chernivtsi	0.593
6	Zakarpatska	1.000	1	Zakarpatska	1.000		
7	Zaporizhzhia	0.504	23	Rivne	0.856	Chernivtsi	0.144
8	Ivano-Frankivsk	0.803	8	Rivne	0.843	Chernivtsi	0.157
9	Kyiv	0.621	17	Lviv	0.144	Zakarpatska	0.856
10	Kirovohradska	0.727	10	Chernivtsi	1.000		
11	Luhansk	0.704	12	Chernivtsi	1.000		
12	Lviv	1.000	1	Lviv	1.000		
13	Mykolaiv	0.674	13	Rivne	0.092	Chernivtsi	0.908
14	Odesa	0.968	6	Lviv	0.988	Zakarpatska	0.012
15	Poltava	0.468	22	Rivne	0.318	Chernivtsi	0.682
16	Rivne	1.000	1	Rivne	1.000		
17	Sumy	0.630	15	Chernivtsi	1.000		
18	Ternopil	0.750	9	Chernivtsi	1.000		
19	Kharkiv	0.611	18	Lviv	0.616	Zakarpatska	0.384
20	Kherson	0.713	11	Rivne	0.082	Chernivtsi	0.918
21	Khmelnytskyi	0.641	14	Rivne	0.426	Chernivtsi	0.574
22	Cherkasy	0.558	21	Chernivtsi	1.000		
23	Chernivtsi	1.000	1	Chernivtsi	1.000		
24	Chernihiv	0.583	19	Chernivtsi	1.000		

Source: authors' own research.

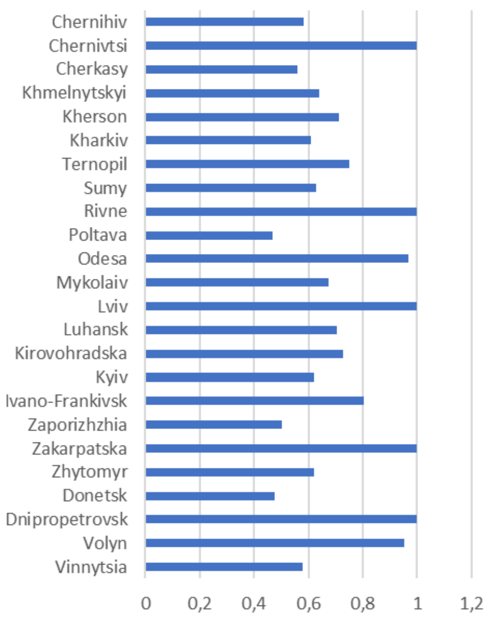

Fig. 1. Efficiency assessment results and reference regions, 2018
Source: authors' own research.

Five regions – Rivne, Chernivtsi, Zakarpatska, Lviv and Dnipropetrovska, according to the calculations, have the highest degree of efficiency and therefore act as reference points for other regions. The coefficients of the reference regions determine the amount of contribution of this reference to the hypothetical object, which will be the target for this inefficient object. For example, for Vinnytsia region the references are Rivne (weighting coefficient 0.869) and Chernivtsi (weighting coefficient 0.131) regions. Since the weighting coefficient for Rivne region is higher than for Chernivtsi region, it means that the structure of Vinnytsia region is closer to the structure of indicators of Rivne region. Table 4 shows that Kirovohradska, Luhansk, Sumy, Ternopil, Cherkasy, and Chernihiv regions are similar in structure to the structure of indicators of Chernivtsi region.

The DEA method forms the recommended values of indicators for inefficient areas. If inefficient areas reach the recommended indicators, they will reach the limit of efficiency and will become effective. Table 5 gives the recommended values of indicators.

Table 5. Recommended values of indicators

No	Name of the region	Efficiency indicator		Results of the DEA modeling					
		Initial values of indicators		Recommended values of indicators		Deviation		Deviation in%	
		Mortality	Birth rate	Mortality	Birth rate	Mortality	Birth rate	Mortality	Birth rate
1	Vinnytsia	**0.579**							
		24341	12769	14100	12769	-10241	0	42.1%	0%
2	Volyn	**0.952**							
		13710	11270	13051	11270	-659	0	4.8%	0%
3	Dnipropetrovsk	**1.000**							
		52336	25121	11270	25121	0	0	0%	0%
4	Donetsk	**0.474**							
		40174	15894	19055,695	15894	-21118.305	0	52.6%	0%
5	Zhytomyr	**0.622**							
		20227	10612	12590,4	10612	-7636.6	0	37.7%	0%
6	Zakarpatska	**1.000**							
		15320	13883	15320	13883	0	0	0%	0%
7	Zaporizhzhia	**0.504**							
		27871	12708	14057,6	12708	-13813.4	0	49.6%	0%
8	Ivano-Frankivsk	**0.803**							
		17449	12645	14013,5	12645	-3435.5	0	19.7%	0%
9	Kyiv	**0.621**							
		28722	15236	17833,374	15236	-10888.626	0	37.9%	0%
10	Kirovohradska	**0.727**							
		15484	7077	11259	8710	-4225	1633	27.3%	23.1%
11	Luhansk	**0.704**							
		15991	5652	11259	8710	-4732	3058	29.6%	54.1%
12	Lviv	**1.000**							
		32726	23253	32726	23253	0	0	0%	0%
13	Mykolaiv	**0.674**							
		17156	9141	11560,7	9141	-5595.3	0	32.6%	0%
14	Odesa	**0.968**							
		33607	23144	32523,518	23144	-1083.482	0	3.2%	0%
15	Poltava	**0.520**							
		23659	10193	12297,1	10193	-11361.9	0	48.0%	0%
16	Rivne	**1.000**							
		14528	13380	14528	13380	0	0	0%	0%
17	Sumy	**0.630**							
		17877	7114	11259	8710	-6618	1596	37.0%	22.4%
18	Ternopil	**0.750**							
		15013	8545	11259	8710	-3754	165	25%	1.9%
19	Kharkiv	**0.611**							
		42600	19657	26045,96	19657	-16554.04	0	38.8%	0%
20	Kherson	**0.713**							
		16163	9095	11528,5	9095	-4634.5	0	28.7%	0%
21	Khmelnytskyi	**0.641**							
		19736	10698	12650,6	10698	-7085.4	0	35.9%	0%
22	Cherkasy	**0.558**							
		20181	8637	11259	8710	-8922	73	44.2%	0.8%
23	Chernivtsi	**1.000**							
		11259	8710	11259	8710	0	0	0%	0%
24	Chernihiv	**0.583**							
		19304	6854	11259	8710	-8045	1856	41.7%	27.1%

Source: authors' own research.

In the process of modeling, four different models of the DEA method were used to obtain broader and more reliable information. Models (1) and (2) are input-oriented, and models (3) and (4) are output-oriented. Models (1) and (3) are CRS models, while models (2) and (4) are VRS models.

The analysis of modeling results allows distinguishing two groups of regions in Ukraine: one with the most effective indicators of the demographic situation, and the second – with the least effective indicators. A comparison of the results calculated by the input models with the results calculated by the output models shows that the groups of regions selected in the study are invariant when using different method models.

Conclusions

The article presents the main theoretical models of the Data Envelopment Analysis (DEA) method for input and output. As an application of the method, the state of the demographic situation in the regions of Ukraine for the ten-year period was analyzed.

According to the DEA method, the indicators of the effectiveness of demographic situations in the regions of Ukraine for 2008–2018 are calculated. Comparative analysis of the obtained results allows to distinguish two groups of sustainable demographic development areas. Zakarpatska, Rivne, Volyn and Chernivtsi regions during the study period form a group with a favorable demographic situation. And Such regions as Chernihiv, Sumy and Luhansk had the lowest level of demographic indicators for the period of 2008–2018.

The calculations obtained in the process of modeling confirm the real condition of the demographic situation in the regions of Ukraine, which is characterized by real statistical indicators of population in the regions. For each region, the research has established the reference regions, the structure of which is similar to the structure of indicators of the set regions. The research also presents calculations of the recommended values of indicators, which set guidelines for improving the demographic situation in each specific region of Ukraine. The prospect of further research is to apply a certain methodology to the data of 2023 to reflect the effects of the pandemic and further forecast the demographic situation in Ukraine.

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