The Riffner National University of the Humanities
School of mathematics and physics
Department of Advanced Mathematics

Qualification work

Masters level

theme:

Application of geometric transformation in solving geometric problems

Completion: Master student group

specialty:

Subject Teaching (Mathematics)

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critic:

content

introduce

The relevance of the subject matter. One of the main challenges of social science and technology and socio-economic development is how to effectively develop, improve and implement information and communication technology (ICT) applications, especially in the field of education. The use of these technologies has greatly improved the efficiency of information processes, including the collection, search, analysis, systematization, summary, processing, storage, and transmission of information and data. Especially in the field of education, whether the perfection of information resource processing method directly affects the quality and efficiency of education.

With the rapid development of information technology, its application in teaching is increasingly extensive, especially in geometry education, geometric transformation as a powerful tool, has great significance for solving complex geometric problems. Geometric transformation can not only simplify the problem, but also help students to better understand the spatial relations and geometric structure. Therefore, the methods of exploring and applying geometric transformations to solve geometric problems have become the focus of educators.

Higher education institutions play a key role in training professionals who can use information technology. These institutions ensure that students have access to quality educational resources anytime and anywhere by adopting distance education technology and modern information technology. In addition, many universities have set up dedicated research teams dedicated to the development and application of information technology in the field of education, aiming to improve the quality of teaching through innovative teaching methods.

In mathematics education, the study of geometric transformation is very important to cultivate students logical thinking ability and spatial imagination ability. Especially in the advanced stage of mathematics education, such as university mathematics education, the concept and application of geometric transformation are more complex and profound. Therefore, an in-depth study of the application of geometric transformation in solving geometric problems can not only help to improve students problem-solving skills, but also stimulate their interest and enthusiasm for mathematics.

subject investigated. Application and effect of geometric transformation in university geometry teaching.

The goal of work. The study aims to discuss the application of geometric transformation in solving geometric problems, including analyzing the basic theory of geometric transformation and its application in different types of geometric problems; discussing how to simplify complex geometric proof and calculation problems through *geometric transformation, proposing effective strategies and methods to solve practical geometric problems, and designing teaching activities based on geometric transformation to promote students understanding and application of geometric transformation.*

Workload: This literature review consists of the introduction, three main chapters, conclusions, and reference lists. The first chapter summarizes the basic theory of geometric transformation; in the second chapter discusses geometric transformation in the design of the teaching activities based on geometric transformation. In addition, the article includes an appendix providing additional supporting material.

Trial of the study: The results of this study have been presented at the 2024 Academic Annual Conference of the Chinese Mathematical Society will be held in Jiaxing, Zhejiang Province from October 31 to November 4, and have been recognized and received positive feedback from peer experts. During the conference, the author also had in-depth exchanges with other scholars that further enriched the content and perspectives of this study.

Adventitia section I

introduction

In todays era of rapid development of science and technology, science and technology and social economy interweave and cooperate together. The booming evolution and application of information and communication technologies in daily practice is both a major opportunity and a key challenge. It significantly improves the efficiency of all aspects of information processing (such as collection, search, analysis, systematization, summary, processing, storage and transmission), and comprehensively reshaping the development path and operation mode of all walks of life. In the field of education, especially in geometry teaching, the integration of information technology is making profound changes.

As the core content of geometry, geometric transformation has a high value in solving geometric problems. Translation, rotation, scaling, symmetry and other transformations, can simplify the complexity, so that the solution of complex geometric problems is more intuitive and efficient. This is not only helpful to solve problems, but also to cultivate students spatial imagination and logical reasoning ability. With the rapid development of information technology, the fields of computer graphics and computer-aided design are constantly making breakthroughs, and the application means of geometric transformation in teaching are becoming more and more diverse. With the help of computer software, students can intuitively feel the dynamic process of geometric graph transformation, and deeply understanding its nature and law. At the same time, the distance education technology is widely used, breaking the limitation of time and space, so that more students can obtain high-quality education resources, bringing new opportunities for geometric transformation teaching.

Research background: Modern information technology has a profound influence on education, and the application of geometric transformation in geometry teaching needs to keep pace with The Times. The current educational reform emphasizes the cultivation of students comprehensive ability. Geometric transformation is very important for geometric learning, but the traditional teaching methods are insufficient in presenting the dynamic and intuition of geometric transformation, and information technology provides the possibility for improvement.

Objective: This study focuses on exploring the application of geometric transformation in solving geometric problems, especially the practical application in the modern teaching environment. Committed to the use of information technology, to build a set of systematic and operational teaching methods system, covering theoretical teaching, practical operation and case analysis and other links, effectively teach the knowledge and skills of geometric transformation, and cultivate students geometric thinking and problem solving ability.

The importance of research: it helps to improve the teaching quality of geometry science, make students better master the knowledge of geometry transformation, and enhance the ability of spatial thinking and logical reasoning. To provide teaching reference for educators, promote the geometry teaching reform, adapt to the development trend of modern educational technology, and promote the effective utilization of high-quality educational resources.

Research questions or assumptions: it is assumed that information technology can significantly improve students understanding and application of knowledge of geometric transformation. The research question includes which information technology presentation method of geometric transformation is most conducive to students learning? How adaptable is the geometric transformation teaching method system under different learning environments and teaching conditions? How to optimize the teaching strategies through case analysis to improve the students geometric problem-solving ability?

1.1 Research Background

1.1.1 Requirements of the modernization of mathematics education

As the basic subject of modern education, with the continuous promotion and implementation of education modernization, mathematics education is also developing to the direction of modernization. In Mathematics Education[1]Zhong, Storiar said that the modernization of education is not to teach students abstract modern mathematics knowledge, but to teach mathematics on the basis of modern mathematics thought, so that the teaching methods and languages of university mathematics are closer to the language and methods of modern mathematics, so as to improve students learning ability. Students mathematical thinking is developing to the thinking direction of modern mathematics. Therefore, to achieve this development goal, the front-line teachers need to re-understand and deal with the content of university mathematics courses. Set theory is the basis of modern mathematics. Using the perspective of set theory to study geometry, that is, using geometric transformation to study figures, which is the method of modern geometry research. In the stage of compulsory education, primary school students do not have the cognitive basis of accepting and understanding the thinking methods due to the imperfect development of various abilities. University is a critical period for the development of thinking. At this stage, students thinking presents a leap-forward development, with a strong thirst for knowledge and the spirit of exploration, and has the basis of accepting new ideas and methods. Therefore, the introduction of geometric transformation thought in university geometry teaching is to realize geometry

The advantageous means of teaching modernization is reflected in the geometry curriculum reform program of all countries in the world. Traditional college geometry courses focus on Euclidean geometry, which understands geometric figures from a static perspective and focuses on cultivating students deductive reasoning and geometric argumentation ability. In 1872, F. Klein put forward the view of using transformed group unified geometry in the comparative view of modern geometry research, namely ″Erlangen Program″[2]. F.Klein and so on The basic idea is that each class of geometry can be described by the transformation groups. The projective geometry is described by projective transformation and studies the invariance problem in projective transformation. Affine geometry is described by affine transformation, which studies the geometry of invariant relations in affine transformation. Euclian geometry is the geometry under contract and similar transformation, and it studies the geometry of invariant properties in contract and similar transformation. This method of studying the invariant properties of graphs with geometric transformation provides a new method for the study of elementary geometry, and also has profound significance for the development of geometry.

Thought is the summary of the essence of knowledge, mathematics educator Misis hidden in the thought, spirit and method of mathematics[3]It points out that many students enter the society to work, but have little opportunity to use mathematical knowledge. But when the mathematical knowledge is forgotten, what is left is the spirit, thoughts and methods of mathematics, these spirits, thoughts and methods will play an important role in a persons life, so that it will benefit all his life. The geometric transformation thought also has such a role, and how can this thought be deeply rooted in the minds of students? This requires teachers to consciously infiltrate these mathematical thoughts in teaching, and students can gradually understand, understand and absorb the thought of transformation.

1.1.2 Requirements of curriculum standards for geometric transformation

In the Curriculum Standards of Compulsory Education Mathematics promulgated in 2001, geometric transformation is introduced into the geometric curriculum system of compulsory education. The geometric transformation in elementary mathematics includes contract transformation and similar transformation, while contract transformation includes translation, rotation and axis reflection. Contract transformation has the characteristics of maintaining constant distance and constant Angle. Transforms in college mathematics textbooks appear in the form of translation, rotation, axisymmetric (folding), etc., and similar transformations also appear in similar ways.

2011 edition of Mathematics Curriculum Standards for Compulsory Education[4]It is emphasized that on the one hand, mathematics teaching should pay attention to the transmission of explicit knowledge to students, on the other hand, mathematics teaching should pay attention to infiltrate the ideological methods behind the knowledge into students learning. Only by reserving the corresponding knowledge and understanding the relevant ideas, can students realize the real value and function of mathematics, and can they think about the world with mathematical thinking. The geometric transformation in the university geometry course can be used as both a tool to understand geometric figures and a geometric idea hiding the essence behind the knowledge of geometric transformation. Integrating the idea of geometric transformation into the process of geometry teaching can not only improve students understanding level of plane geometry knowledge, cultivate students dynamic geometry concept and geometric thinking, and then improve students ability to solve plane geometry problems.

The compulsory education model in Shanghai is different from the June 3th system in other parts of China, but the May 4th system. Shanghai has formulated an independent curriculum standard, ″Shanghai Mathematics Curriculum Standards for Primary and Secondary Schools″[5](Hereinafter referred to as the ″Curriculum Standards″). In the part of graphics and geometric content, students are required to understand the basic geometric transformation, describe the movement law of graphics with geometric transformation, and timely carry out two special courses of axisymmetric transformation, which shows the application value and important position of geometric transformation in the mathematics course of compulsory education in Shanghai. As far as the compilation of the textbooks is concerned, the textbooks of the Beijing Normal University edition adopt the spiral mode in the course arrangement of geometric transformation, and set the content of geometric transformation separately in the textbooks of all grades. In speaking, the Shanghai edition of the textbook is unique, setting ″graphic movement″ in the last chapter of the first semester of the seventh grade. The teaching content includes translation, rotation, basic concepts and basic properties of symmetry, as well as basic geometric mapping. This is the first time for college students to be exposed to the geometric content. After a careful study of the teaching materials, we found that this arrangement has laid a foundation for the university geometry teaching based on the theory of geometric transformation, which is embodied in the teaching materials. For example, when the textbook introduces the concept of complete equality, the movement of the figure is used. The nature teaching of isosceles triangle is to use folding to feel the symmetry of isosceles triangle, and to inspire students to add the auxiliary line for geometric argument, and the addition of the auxiliary line in the proof of the theorem is essentiallyRotation transform and so on."Standard" requirements through the movement of graphics make students preliminary perception of geometric transformation, students can understand geometric transformation, auxiliary add essence, and some questions from the perspective of geometric transformation will become easy, geometric transformation for geometry teaching and learning to provide more operation space, students can explore through geometric transformation and put forward new conclusions, and deepen the students of geometry

1.1.3 The actual situation of university geometry teaching

Through communication with practice school teachers, learned that some teachers reflect the seventh grade first semester will translation, rotation, axisymmetric three geometric transformation, and the seventh grade second semester and eighth grade is all about the content of geometry, no about the content of geometric transformation, until the ninth grade first semester and similar transformation, such a long time span of teaching makes geometric transformation teaching effect is not ideal. On the one hand, students have difficulty in learning the geometric transformation plate, especially in the understanding of rotary drawing; on the other hand, the application value of geometric transformation in other aspects is not really reflected, and students naturally cannot realize the application value of geometric transformation. The root of such geometry teaching is that some university mathematics teachers do not have enough understanding of geometric transformation, which should also be one of the reasons why geometric transformation has not been paid attention to in teaching.

Through correcting students homework during educational practice, it is found that some ninth grade students understanding of geometry only stays on the surface, which is reflected in the simple thinking of geometry problem solving, and the "fear of difficulties" for geometric comprehensive problems. Through communication with teachers to understand the ninth grade will special training for the movement of the graphics, but students are difficult to figure the movement of the movement in the exam of geometric comprehensive answer students still lose points, students comprehensive analysis ability is not really improved. The common problem of students is that they can understand the static geometric figures, and it is difficult to understand the movement of the graphics. There is no idea of movement transformation in their mind, and they cannot make the graphics after translation, rotation and folding, and the graphics are the key breakthrough to solve the geometric problems.

In a word, such a current situation of geometry learning seems to be due to the lack of motion transformation, but its essence lies in the teachers failure to realize the importance of geometric transformation, especially failed to penetrate geometric transformation as an thought method in teaching. On the one hand, the formation of students ideas is the result of individual physical and mental development, and on the other hand, it depends on the conscious cultivation of teachers in the teaching process. In order to improve students ability to view geometric problems from the perspective of motion transformation, it is necessary for teachers to permeate the idea of geometric transformation for a long time in the process of geometry teaching.

1.2 Research questions

Through the above relevant background analysis, the development of geometric curriculum modernization requires the application of geometric transformation; the teaching of mathematics curriculum standard requires not only mastering the knowledge of geometric transformation, but also using geometric transformation as a tool to understand the nature of geometric figures and a method of learning geometry. In the teaching of geometric transformation thought method of penetration depends on teachers, because the university teachers understanding of geometric transformation, and the understanding of geometric transformation concept of geometric teaching is not enough, students use geometric transformation to understand geometry concept, theorem and geometry problem solving is not ideal, students comprehensive analysis ability and geometric ability to explore also need to improve. Ninth grade geometry learning is not only for the deepening of university geometry learning, but also the foundation of high school geometry learning, research shows that ninth grade students van hill geometry thinking level generally reached level 2 or level three, has developed to can accept the infiltration of the corresponding mathematical thought conditions, so through the ninth grade students geometry transformation thought infiltration teaching practice to improve the ninth grade geometry teaching status has the value of research and research is necessary. Educational research comes from educational practice, which is based on the actual teaching situation. In this paper, teachers and students know several universities through questionnairesWhat geometric transformation thought penetration and application of teaching situation, and further study the carrier of geometric transformation thoughts penetration, according to the actual situation of students in the corresponding teaching design and teaching practice, in order to improve the students geometry learning situation, and cause the attention of the teachers of geometric transformation thought penetration, provide a line of geometric teaching teachers teaching

This paper mainly studies the following four problems:

1. What is the penetration and application of the idea of geometric transformation in university geometry teaching?

2. What are the effective methods to permeate the idea of geometric transformation?

3. What are the promoting effects of the teaching of the idea of geometric transformation on the geometric learning of grade 9 students?

4. For students of different levels, are these promotion effects somewhat different?

1.3 Study Methods

(1) Literature research method

By consulting relevant literature and books, clarify the concept of geometric transformation, determine the theoretical foundation of education and teaching in this paper, lay the theoretical foundation of research; comb the existing research results of geometric transformation, analyze the previous research experience and find out the innovative points, and clarify the research ideas and research framework.

(2) Questionnaire survey method

The early stage of the study was conducted through questionnaire survey, the purpose is to understand the current situation of "teaching" and "learning" in university geometric transformation, and the questionnaire survey for teachers is to understand the understanding of geometric transformation and the penetration of geometric transformation in teaching; the questionnaire survey for students is to understand the understanding of geometric transformation and geometric transformation, and the current situation of teaching and learning provides a certain realistic basis for the development of this study.

(3) Quasi-experimental research method

Teaching practice through the design of geometric transformation thought of geometric teaching quasi experimental research, in order to control the interference of irrelevant factors, participate in the experiment of the class selection standard is mathematical parallel class, through the ninth grade examination and similar triangle unit test result analysis, finally chose the practice school a 15 years teaching age of mathematics backbone teachers with two level of class, including nine (2) classes for experimental classes, nine (12) for the control class. In addition to the teaching intervention of the experimental class in the teaching of geometry that permeates the idea of geometric transformation, the rest are completely arranged according to the teaching schedule of the school. Teaching intervention includes introducing geometric transformation ideas, using geometric transformation understanding geometric figures, using geometric transformation understand similar geometric model, using geometric transformation add auxiliary line, using geometric transformation geometric development and explore, using geometric transformation geometric art design extracurricular activities, etc., to reflect the permeability geometry transformation of geometric teaching, before the experiment of students questionnaire and geometric test as the basis of the experimental results.

(4) Work analysis method

After the experiment, on the one hand, the cases of the geometric test results of the experimental class students were compared to compare the ability of students with different grades in geometric exploration, and the influence of the penetration of transformation ideas on the geometric exploration ability of research, and the difference of the acceptance of students geometric transformation ideas. After the experiment, on the other hand, through the analysis of students geometric problem solving thinking flow chart, the analysis of students thinking, study the influence of the teaching experiment of geometric transformation thought infiltration on the development of students thinking ability.

1.4 Study purpose and study significance

(1) Study purpose

The purpose of educational research is to improve teaching and then promote the all-round development of students. The new educational concept needs the attention of the front-line teachers and put it into practice, so as to continuously promote the development of education. The infiltration and application of geometric transformation thought in university geometry teaching is a preliminary attempt to infiltrate modern mathematical thought in the university stage; on the other hand, it causes the frontline teachers to pay attention to the infiltration of geometric transformation thought, improve the students geometry learning, lay the practical foundation for the infiltration of geometric transformation thought in teaching, preliminarily explore the feasibility of establishing plane geometry teaching on geometric transformation theory, deeply integrate the European geometry with geometric transformation, and make the preliminary transformation of plane geometry with geometric transformation.

(2) Research significance

Teachers are in the core position in teaching activities. Teachers knowledge base, the depth of knowledge understanding and the teaching concept all directly or indirectly affect students learning concept and learning depth. So teachers should have the high view of modern mathematics, teachers should further study the curriculum and teaching knowledge structure, mining knowledge internal connection, attaches great importance to the ideas behind the knowledge, help students on the basis of the original cognitive to establish new knowledge structure, deepen their understanding of knowledge, and constantly cultivate their ideological quality.

1. Change students ideas and ways of geometry learning

Geometrical transformation is widely used in various fields of modern society. Only standing in the perspective of modern mathematics to understand geometry, understand the idea of geometric transformation, and apply it to geometry learning, can students constantly cultivate the concept of movement transformation, experience the invariable dialectical thought in the transformation.

2. To provide reference for improving geometry teaching

Teachers want to improve geometry teaching through infiltration geometric transformation, improve the teaching quality of the geometry, the first thing to have a reference to teaching material and teaching case, this study can arrange the teaching material of geometric transformation thought of knowledge, examples, exercises and extracurricular activities related to the carrier, and design the teaching case based on geometric transformation of inquiry, for the teachers teaching practice provides a reference of teaching design case.

3. It provides a new direction for the teaching reform of plane geometry

The traditional geometry teaching method should be transformed to form the geometry teaching mode based on inquiry, so that students can independently explore the geometry knowledge, change the students understanding of geometric static, and form the geometric view of motion. On this basis, cultivate students geometry topic research and preparation, improve students various ability, make the transformation of teaching not only reflected in the results, pay more attention to the operation and process, through the teaching of geometric transformation of operation and process, reflect the education value of geometric transformation, to transform university plane geometry teaching provides a new direction.

4. Promote the close combination of modern mathematics and university mathematics

The significance of geometric transformation is not only that it can help solve some difficult geometric problems, but more importantly, it is closely related to the concept of matrix and transformation group in modern mathematics, which is described by matrix in high school geometric transformation. Geometric transformations are widely used in the processing of digital images, and group theory is also widely used in social and scientific development, such as group theory describing the symmetry of molecules and crystals in the field of chemistry. Therefore, it is very necessary to infiltrate the idea of geometric transformation into university mathematics teaching. To introduce the idea of transformation and group theory to students at appropriate times, so that students can understand the concept of modern mathematics and create conditions for students to further study. The penetration of modern mathematical thought in university, can only be subtle, and as a university mathematics teacher, on the view of geometric content to a little higher, deep, only in this way, to commanding the university geometry teaching, shorten the distance between university mathematics and modern mathematics, create a new situation of modern geometry teaching.

1.5 Research ideas and research framework

First of all, through the analysis of the existing studies, most of the existing studies understand the current situation of university geometry teaching only from the perspective of teachers or students, and the current situation of geometry teaching includes both teachers "teaching" and students "learning". Based on this, in the current situation survey stage of the early stage of the study, through the questionnaire of teachers and students, all the mathematics teachers in the practice school were selected as the object of the questionnaire survey. A total of 33 teachers included 3 retired teachers. Ninth grade students have learned most of the university geometry content, so this paper selected 212 ninth grade students about geometric transformation learning questionnaire, and select two of the experimental class for geometric test, test students use geometric transformation of geometric problem solving, to understand the college students understanding of geometric transformation and use of the reality. After understanding the current situation, the problem to be considered is which carriers in the teaching can penetrate the idea of geometric transformation. This paper combs and sorts out the carriers of penetration. After sorting out the corresponding carrier, it is necessary to consider what principles and effective measures should be followed for penetration. This paper combines with existing studiesIn the teaching principles and teaching strategies of mathematical thought methods, combined with the particularity of geometric transformation thought, the corresponding teaching principles, teaching objectives and teaching measures of geometric transformation thought infiltration are formulated. Finally, the teaching design and quasi-experimental study of the idea of permeant geometric transformation were carried out, and the control experiment was designed. After the teaching experiment, the experimental results were measured and analyzed from many aspects, and the experimental results were compared. The questionnaire survey was used to understand the influence of the infiltration experiment on students non-intellectual factors such as the understanding of geometric transformation. The statistical results of the five math scores before and after the experiment are used for quantitative analysis to analyze the effect of the infiltration experiment in improving the students math scores. At the same time, through the case comparison of the geometric test results of the experimental class, the analysis of students with geometric transformation in geometric exploration, so as to show the acceptance of students with different grades of geometric transformation. Using the method of "loud thinking", the students solve mathematical problems while thinking, record the thinking process, draw the flow chart of the organized thinking process, and analyze the thinking process according to the flow chart of "loud thinking". Finally, the experimental results. The study of thisThe study framework is shown below in Figure Figure 11-

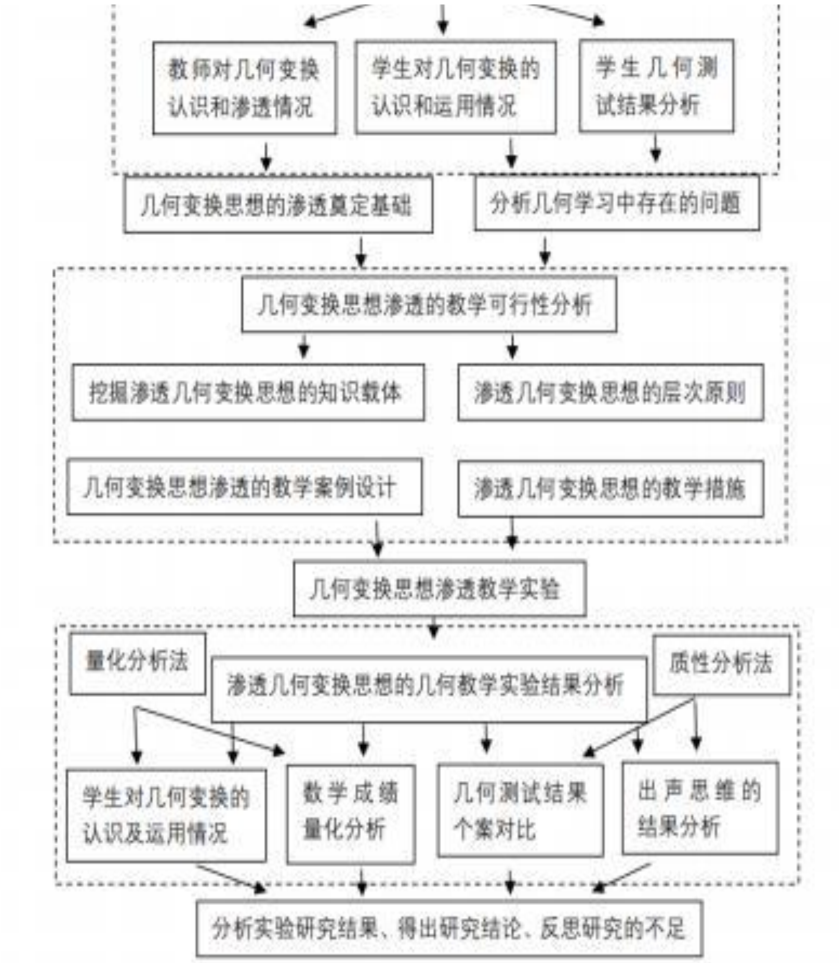


Figure Figure 1-1 The study framework

Adventitia section 2

Research review and theoretical basis

2.1 Definition of core concepts

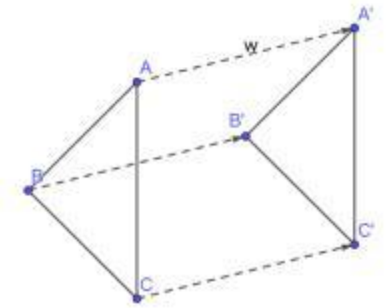
2.1.1 Geometric transformation

Any graph F on the plane can be regarded as A set of points. If every point A on the graph becomes another point A in the same corresponding rule T, then all A form another new graph F in the plane. This rule T from graph F to graph F is called geometric transformation, denoted as T (A) =A , T (F) =F. In college, the main geometric transformation include contract transformation and similar transformation, in which contract transformation has translation, rotation, and axisymmetry, which can be obtained through the joint action of bit likand contract transformation. This paper defines the concept of geometric transformation, that is, the elementary geometric transformation proposed in the Shanghai Mathematics Curriculum Standard for Primary and Secondary Schools, which includes translation, rotation, axial symmetry (folding) and similarity in university mathematics textbooks.

2.1.2 Common elementary geometric transformations

(1) Translation transformation

Moving all the points on the graph F for the same length in the same direction is called the translation transformation, for translation. Where the direction of the translation is the direction of the movement, and the translation distance is the length of the movement. A translation transformation is determined by a vector, and the direction and mode of the vector determine the direction and distance of the translation, respectively.(As shown in Figure Figure 2-1)



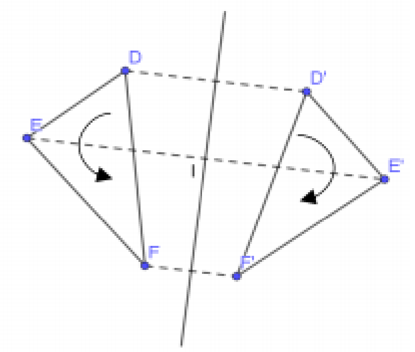


Figure 2-1 Translation Figure 2 - 2 Axisymmetric

(2) Reflection transformation (axisymmetric)

There are two graphs F and F in a plane, which are axisymmetric, and line l means the symmetry axis of F and F. If the graph F is transformed into graph F , this transformation is called reflection transformation, line l is called the reflection axis, and the turn of F and F is opposite, so it is called a mirror contract.(As shown in Figure Figure 2-2)

The reflection transformation has the following properties:

①Transform graphics into equal graphics;

②For two points in a straight line, the connected segment is divided vertically by the line.

(3) Rotational transformation

For each point on the graph F, rotate the same point O clockwise (or counterclockwise) to the same angle A to get the graph F . The transformation from the graph F to the graph F is called the rotation transformation, O is the center of rotation, and angle A is the angle of rotation.(As shown in Figure Figure 2-3) A rotation transformation is determined by the center of rotation, and the direction of rotation.

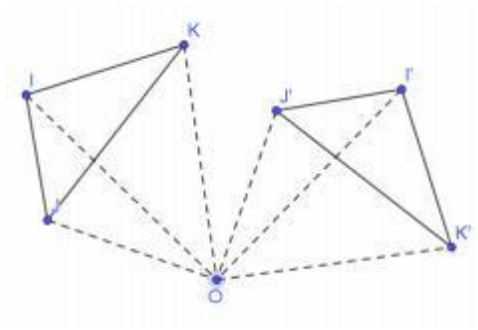


Figure 2-3 Rotation

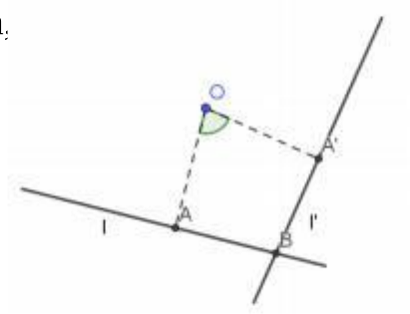


Figure 2-4 Rotational properties

The rotation transformation has the following properties:

①Two figures before and after rotation.

②The angle of the corresponding line segment connected to the center of rotation is equal to the rotation angle.

③If you put the line l, center on the point O not on it, rotate the angle in the specified direction

A (0° <a <180°) gives the line l , then the angle between the line l and the line l is equal to the rotation angle a.do. Al at point A, as a. Al ′ and point A ′ (as shown in Figure 2-4)

Note that line l intersects line lat point B.

Because A, O, A , B four round,

So <l, l > =a.

That is, the clip angle between the line l and the line lis equal to the rotation angle a.

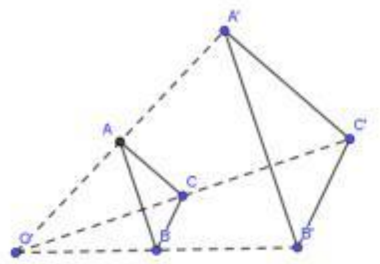
(4) Bit-like transformation

There are two graphs F and Fin the plane, and there is a one-to-one correspondence between the points and points, satisfying the following two properties:

①Any straight line connecting a pair of corresponding vertices goes through the same point;

②If A and A are any pair of corresponding points, there is OA = | K | OA.

Then the figures F and Fare called bitlike graphs. Changing a graph F to a transformation of a graph F , as described above, is called a bit-likelihood transformation.



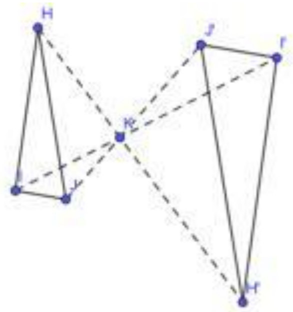


Figure 2-5 bit likelihood (k> 0) Figure 2-6 bit likelihood (k <0)

(5) Similar transformation

If the graph F is a bit-like transformation H and a contract transformation C, then the transformation of the product of the two transformations C and H is called the similarity transformation, which is expressed by S, that is, S = CH.

2.1.3 Ideas of geometric transformation

Geometric transformation thought refers to the graph on the plane after translation, rotation, axisymmetric, similar transformation of one or several kinds of the transformation, the quantity of the figure remains the same idea, such as after the contract transformation of the graph, the length of the corresponding segment, the angle is the same, the size of the corresponding angle, the ratio of these movements are collectively referred to as the geometric transformation idea.

2.1.4 Perenetration of geometric transformation

Penetration means that some thing or thought gradually enters into other aspects. In this paper, the infiltration meaning of geometric transformation thought in this paper is that the idea of invariant implied in geometric transformation gradually enters into other aspects of the geometric teaching process. For example, the idea of geometric transformation permeates into the related teaching of circles, the addition of geometric topics, the exploration of geometric comprehensive topics, and the use of some extracurricular activities. The purpose of penetrating the idea of geometric transformation is to make students gradually use the idea of geometric transformation in geometry learning, understand the modern mathematical thought, and lay the foundation for the study of matrix and transformation in high school and university.

The two ab vectors are also called equal, ıf they meet the following condıtıons: ⟶

(A) Collinear and parallel:; ab ⟶⟶

(B) has the same module (the same length): a= b point 2.2.6. Collinear vectors with the same length but different orientations are called the opposite vector. For the opposite vectors, markers are used. Examples of opposite vectors are the vector and in A, B are arbitrary (different) points of space. |⟶| |⟶|

A, a ABB A at poınt 2.2.7. (A + b) abc The sum of the t

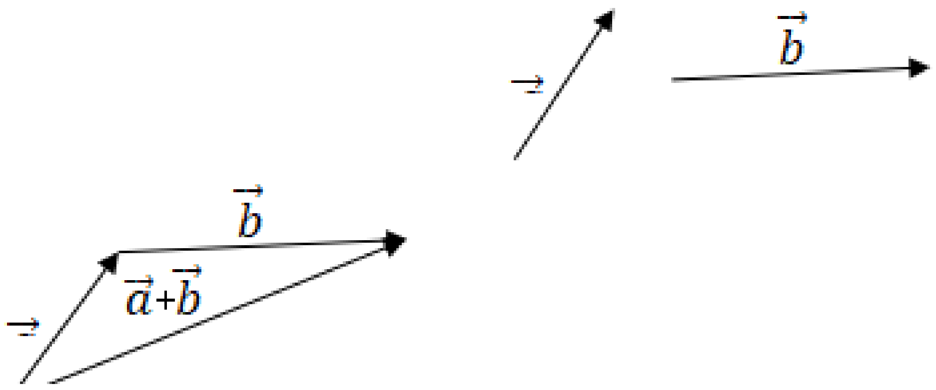


Figure 2.2.2: Triangle rule

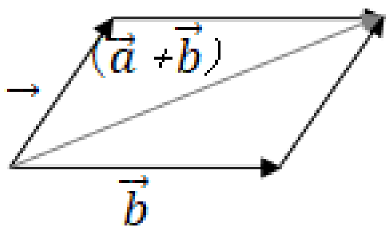


Figure 2.2.3: Parallel ogram rule

Therefore, let two arbitrary vectors I (Figure 2.2.2-2.2.3).a b a1+...+ a n a1,..., anc note 2.2.8. The sum of n vectors is called such vectors whose start matches the start of the vector and the end matches the end of the vector, provided that the end of the vector matches the start of each vector.a i a n a i ai+1i=1,...,n−1

A b a b c b The dıfference between two vectors, wıth such a vector, whıch wıth a sum o

$$a - b = a + -b;$$

(e)∀ a∃ a′= a: a+ a′= o;

$$|a + b \leq a + b, \quad a + b \geq a - b$$

The multiplication of a (real number) number on the vector is called the vector that satisfies the condition: α = Oa α b= α a $> 01); 2). <O | | = | | | | | $ Theorem 2.2.1. For any vector I and any real number, there is an equal place: ab α, β a) l a= a;; Ia= ab) α a= β aa=0 α = β.

c)α· a| = | α| · | a| ;

d) α · β · a = β · α · a = α · β · a;

e)α a+ b=α· a+α· b;

f)α· a+β· a=(α+β)· a.

α, β R let. Then for any vectors, and with the following properties a, bc

1) a+ b= b+ a ;

2) a+ b + c= a + b+ c ;

[u − P

c)α· a| = | α| · | a| ;

α · β · a = β · α · a = α · β · a;

e)α a+ b=α· a+α· b;

f)α· a+β· a=α+β· a.

α, β R let. Then for any vectors, and with the following properties a, bc

1) $\vec{a}+\vec{b}=\vec{b}+\vec{a}$;

2) $\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)$;

$3)\exists\ o: \forall\ a\ a + o = a,$

$4)\forall\ a \exists\ a = a : a + a = o;$

2.1 Apply the geometric transformation to deduce the proof problem

2.1.1 Geometric transformation of the plane

(1) Drawing transformation

Let the FDA figure be on the plane. If each point in Figure F is somehow arranged and the X points are arranged as the corresponding point X ′ , then we get a new Figure F ′ . The figure is said to be formed by the transformation of the data, and in this process, the X point is called the image of the X point, and the X point is called the image of the Xpoint.

The transformation can be the same (each graph is converted into its own), and is mutual. In the single-value conversion between them, motion plays an important role.

(2) Mobile

Convert one drawing to another, if it keeps the distance between points, converting any two point X in the first drawing and any point in the first drawing to point X and U in the second drawing, making hu=huu (figure).

Note: The concept of motion inconventional concepts geometry is related to
About moving. But if it comes to displacement, we imagine a continuous process, then in geometry, only the start and end positions of the graph will make sense to us.

Two consecutive actions, and gave another action.

Let Figure R into Figure R transform different points of Figure R into different points of Figure R. Let any point in X in Figure P move down by this transformation to the point in X in Figure P. Figure P is converted to Figure X, where Figure Xis at point X

Movement, called the reverse transformation, preserves the distance between points and therefore transforms different points into different.

Obviously, when converted into movement, there is also movement.

kinetic characteristic:

theorem:

During movement, the straight dots are moved to the straight dots, keeping the order in which they are placed to each other.

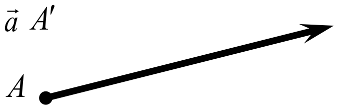
A, B, C A_1, B_1, C_1 This means that when the point lying in the line moves to the point, the points are also in the line; if point B is between points A and C, the point is between points A and C. B_1 A_1 C_1

Derdeduced from the theorem, during motion, lines, lines, lines.

During the movement, the angle between the half-lines is retained.

(3) Parallel transfer.

Parallel transfer, or vector transfer, is called such a plane transformation, where any point A is shown on point A, such that A ′ =. $\vec{a}$ $\vec{a}$



Point A in parallel transfer is vividly called point A.

Introduce the Cartesian coordinates x, u.

The transformation of Figure P, where any point (x; u) transfers to a point (x + a; u), where A and b are identical for all points (x; u), called parallel transfer. Parallel transfer is given by the formula:

$x' = x + a$, . $y' = y + b$

These formulas represent the coordinates (x , u) of the point of parallel transfer (x; u).

Parallel transfer properties

Parallel transfer is a movement.

2. The name "parallel transfer" is due to parallel

The moving point moves at the same distance along a parallel line (or coincidence line).

$AA'B'B$ AB $A'B'$ Note that the two sides in the parallelogram are parallel and the other two opposite sides are I. It is deduced that when transferred in parallel, the line moves to a parallel line (or inside it).

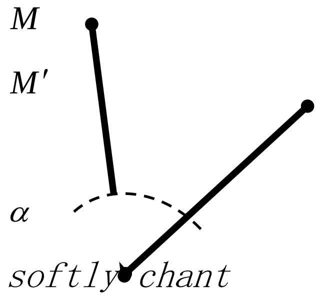
All parallel transfers of the multiple constitute a group.

Theorem (the existence and unity of parallel transfer):

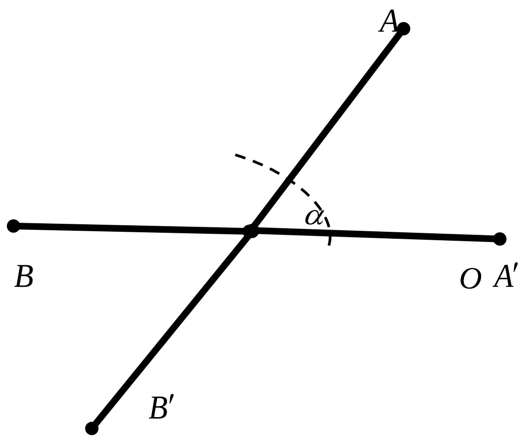
Whatever are of two points A and A , there is an identical one, before which point A is moved on point A.

(4) Turn

α Let the O-point and the orientation angle be in the plane. Given the plane of each point of m, we will be placed at such a point to and. This correspondence is called the O angle of plane rotation around the O point is called the center of rotation, and the Angle is the angle of rotation.. $M' \; OM = OM' \; MOM' = \alpha \; \alpha \; \alpha$



α A turn is a movement. Indeed, if O is the center of rotation, is the angle of rotation, A, A and B, B are the points corresponding to the two pairs, then defined as O a = Oa , O a = Ov. Furthermore, a ov = aov and, a v = av.



In this way, the rotation makes each figure equal to her figure. To build some direct image, just choose any two points above, build their image, and connect directly. To build a circular image, we must build the image

Its center and, centered on it, are at a circumference of the same radius.

The image constructing a given polygon boils to the rotation of its vertices.

In addition to the "direct" task of rotation using the rotation center and rotation angle, an indirect method of assigning values to the two corresponding points should be indicated. There is a theorem.

theorem:

If given two equal non-parallel sections AV and A ′ ′, there is a rotation that transforms the section AV of A ′ ′ to A ′ and B to B ′.

In the case where the AB and A V cut planes are parallel and in opposite directions. In this case, the center of rotation is the intersection of direct AA ′ and BB ′, with a rotation angle of 180°.

If the same cuts of AB and A ′ are equal, parallel, and equally oriented, then the simultaneous translation of A to A and B is absent, but there is a parallel transfer to perform this conversion.

In this way, any movement of the cutting in the plane can be accomplished by rotation or by movement.

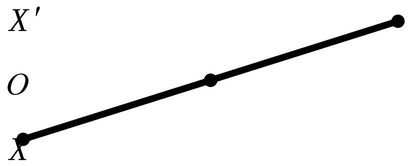
Without elaborate, note that for the other numbers, the last sentence is generally unfair.

Rotation works as a way to solve the geometric construction problem.

The idea of the rotation method is to return an appropriate selection center near any data or search graph to facilitate the analysis of the problem or even to derive solutions directly.

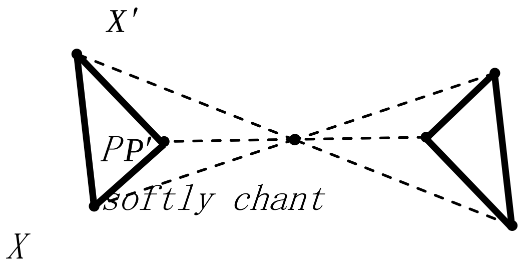
Relative point symmetry

Let the O-fixed point and X be arbitrary points of the plane. We delay the extension of the cut point O of O, equal to the cut point O of O. Point X ′ is called the symmetry point X relative to the O point, point o, is the O point itself, it is obvious, the symmetry point X ′ , is the symmetry point X ′ , is the X point.



Convert figure R to figure R, where each of its point X is superconverted at point X ′ , symmetric

Relative to the given O point, is called symmetric transformation relative to the O point, where graphs R and R are called symmetric transformations relative to the O point.



If the transformation of point O with relative symmetry transforms the graph R itself into a central symmetry, point O is called the symmetry center.

For example, the parallelogram is a centrosymmetric figure. Its symmetry center is the intersection point of the diagonal lines.

theorem:

The transformation relative to the symmetry of the point is a motion.

(5) Central symmetry

O $\alpha = 180^{\circ}$ *A rotation at a center angle is called centrosymmetry.*

The center of rotation is here called the symmetry center.

M' M O M' M O $\angle MOM' = 180^{\circ}$ $|MO| = |M'O|$ If the-dot image is in the centrosymmetric center, then the three dots,,, lie on a straight line because. In addition, because of the central symmetry, as a turn, it is a displacement. We derive the following rule of point image structure under the central symmetry: we make a line on the ray added to the whole line, delay from the section, and so on. M' M

O MO OM O OM' $|MO| = |M'O|$

Centrrosymmetry is marked by centrosymmetry. She is completely O
Z_0
O M' M Center assign or assign a pair of corresponding points and. In the latter case, the switch center will be the center of the incision. MM'

Center symmetry is a rotation whose straight line moves in its direction, the triangle to the triangle, and the direction of the triangle is maintained. The beam and the corresponding beam are in opposite directions. AB $A'B'$

The composition of two symmetries with common centers is a transformation in which each graph transforms it into itself. O F

The geometric problem solving method based on center symmetry application is called central symmetry method.

(6) Symmetry is relatively direct

Let the l-fix the straight line. Take any point X and subtract the one perpendicular to l. At the extension perpendicular to point A, we delay a section AH ′, equal to the section of AH. Point X ′ is called a symmetric point of X with respect to straight l. If the X point is on a line l, then its symmetry point is the X point itself, obviously, a point symmetric with the Xpoint is the X point.

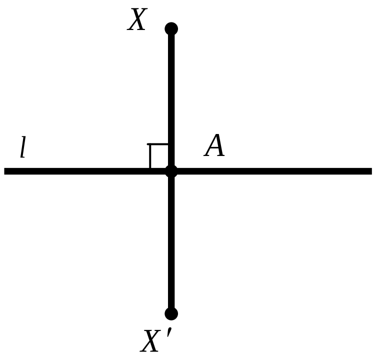
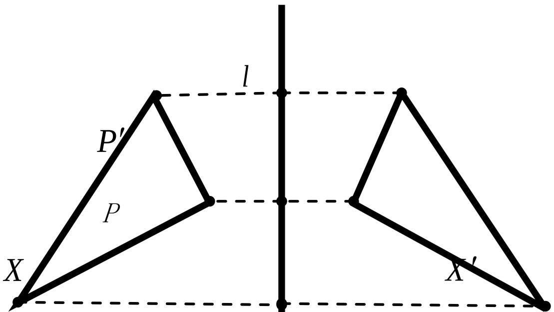


Figure R is converted to Figure R, in which each of its X points is crossed with a given straight 1 symmetrical X point, called a symmetry transformation of relatively straight 1 symmetry (axisymmetric).

In doing so, Figures R and R are called symmetrical relative direct 1.

If the symmetry transformation of the relatively straight 1 transforms the graph R to itself, then the graph is called the symmetric relatively straight l, and the straight l is called the axis of symmetry of the graph.



For example, the straight line with the intersection of the diagonal of a rectangle parallel to its sides is the axis of symmetry of the rectangle. The straight line on which the diamond diagonal lies is its axis of symmetry.

theorem:

A relatively direct symmetry transformation is a motion.

(7) Homogeneity

In the study of parallel transfer, axisymmetric, and plane geometric transformations such as rotation, unify them into a common concept —— movement, or motion.

The plane displacement is known to maintain the size of the geometry and thus the shape of the geometry.

However, there are such plane transformations where the size of the figure changes but the shape remains. This is so, for example, in making a sketch of detail, in drafting a plan of a terrain, in drawing a topography, while making a picture on a movie screen, and while making a photocopy.

In this case, the figure and her image are no longer equal, but have the same form. It is said that the two figures are similar, and in this process, the transformation of converting one figure into another figure is called a similar transformation, or simply called similarity.

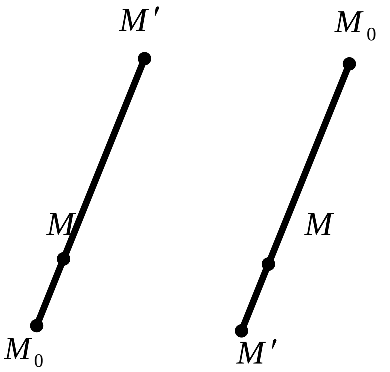
Consider first a separate similarity case, called central similarity, or homomorphism.

Homophone comes from the Greek word homos is the same (similar), thetos is similar, so homophone is identical (similar) homonym.

The nature of homology.

*Center m homology₀*The sum coefficients are called plane transformations, where plane M arbitrary points are corresponding to that point, thus performing a relationship $k \neq 0$ M' $\overline{M_0M'} = k\overline{M_0M}$

$\overline{M_0M'} = k\overline{M_0M}$ The point m is derived from Eqs$_0$, M and M belong to a direct. In doing so, if> 0, the vector and the points of m and m-its image-lie on one side of the center of mk $\overline{\mathrm{M_0M}}$ $\overline{M_0M'}$,homomorphism. In this case, homomorphism is called additivity, or first class homomorphism.



k $\overline{\mathrm{M_0M}}$ $\overline{M_0M'}$ If <0, the vector and direction are opposite, the points of m and m ′ are located on different sides of the center of the homomorphic state. In this case, the homomorphism is called homomorphism, or the second type of homomorphism. The point M in the homomorphism is called the homomorphic point M

The center is m$_0$ k . *And a coefficient is written as follows: OR* $H_{M_0}^k(M) = M'$ $H_{M_0}^k : M \to M'$

k If> 0, the homomorphic center is called external, and if <0, it is called internal. k

Based on the definition of homomorphism, several properties can be established. Lets stop some bit.

1. In homomorphism, the homomorphic center is self-mapping, i. e., invariant points. $H_{M_0}^k(M_0) = M_0$

2. $k \neq 1$ If, then there is no invariant point in the homology, if, then any point of the plane M is invariant, that is, the homology in this case is the homology transformation. $k = 1$

3. $H_{M_0}^k$ If points M and N indicate points M and N, respectively, then $\overline{M'N'} = k\overline{MN}$

$H_{M_0}^k$ If point M, N indicates M ′ , respectively, then. $|M'N'| = |k||MN|$

$H_{M_0}^k$ When keeping a simple three-point relationship.

6. Homomorphic transforms directly into a line parallel to it.

7. The straight line passing through the center of the homomorphism moves in itself, that is, invariant.

8. Set of all homomorphisms with a given center, m_0And form the exchange groups with different coefficients.

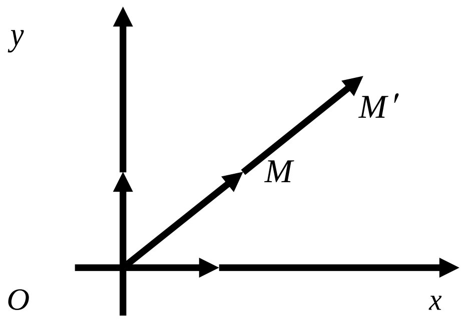
theorem:

Discrete homomorphism of the center m_0And the coefficient k <0 can be expressed as the composition and m point mapping of additional homomisms with the center and the same coefficient $|k|$ *0,*

$H_{M_0}^{-|k|} = Z_{M_0} \circ H_{M_0}^{|k|}$

Analytic expression for the homomorphism.

We will consider two cases:



starting point of the coordinates, namely, m_0=0. m is any point in the plane that intersects the m point on homotopy, where the m point has coordinates (x, u), m ′ (x, u).

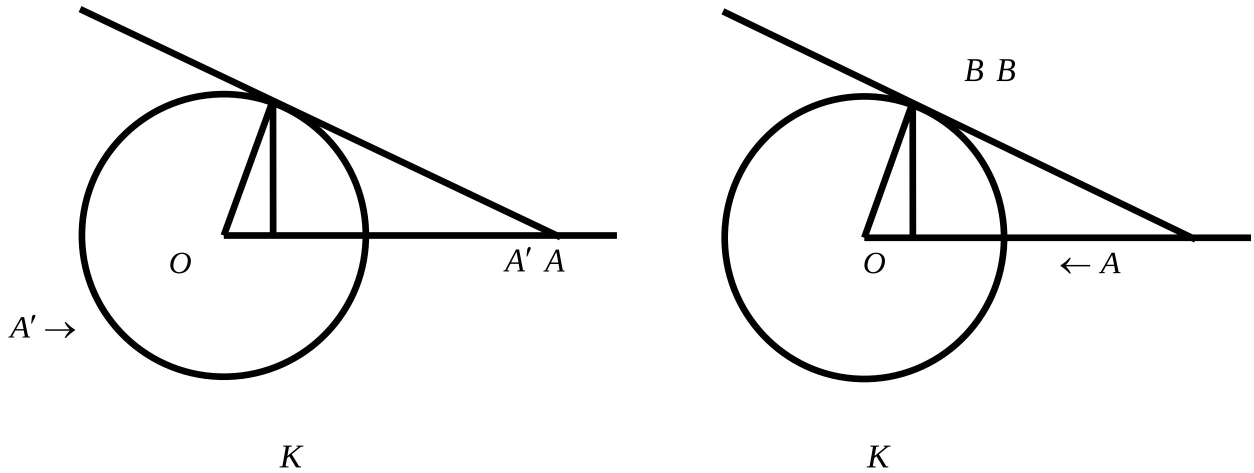
1. Set the M homomorphism H_0^k center$_0$Coincident with the

2. Then, at the beginning of the coordinates and the center is homomorphic, the relationship between the image and the graph coordinates is as follows: $\begin{cases} x' = kx \\ y' = ky \end{cases}$.

Let the point m_0Does not coincide with the starting point of the coordinate system and has coordinates (x_0 , $_0$). M and M have coordinates (x_0, $_0$ *(X, U)* 。

Then: any point on the coefficient and the central homology problem analytical formula $\begin{cases} x' = kx + (1-k)x_0 \\ y' = ky + (1-k)y_0 \end{cases} - k$

Adventitia 8. Reverse



π K r O A In the plane, take the radius of a center at the point. Place each point different from the center of the circle at a position corresponding to the point lying flat on a beam, so that. The point is called with the inverse of the point relative to the circle. Obviously, it is not difficult to construct a point: if the point is located outside the circle, then by contacting the circle from it, we conduct from the contact point perpendicular to the beam. This base is vertical and will be a point. When the point is inside the circle, we establish a perpendicular to the point - contact with the circle. The intersection of this tangent with I will be reversed with that point. If it is on a circle, then the point itself coincides with the point of its opposite. A' OA $|OA| \cdot |OA'| = r^2$ A' A K A' A K B OA A' A K AB OA B OA A A K A A'

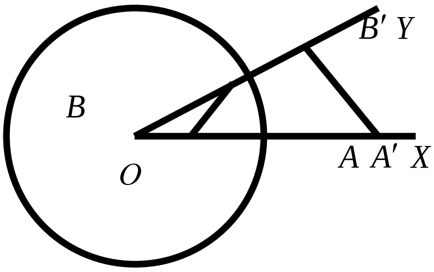
A The transformation that corresponds to each point of the circle and its antipoint is called an inversion. The point is the center of the reversal, the circle K is the reversal circle, and the square of the reversal circle radius is the degree of the reversal. A' O

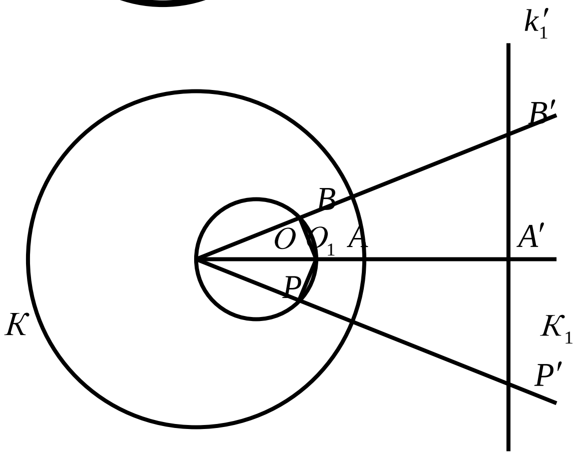
The properties of the inversion:

1. Φ' Φ Φ When a figure is an image when a figure is inverted, it can be asserted that it is an image when the same figure inversion. Φ'

2. OX,OY Take two rays and the inverse points on them, respectively A',B'

A B. Points, actual ratio: $|A'B'| = \frac{r^2|AB|}{|OA||OB|}$.





3. K_1 The circle passing through the inversion center moves in a straight line during the inversion.

K_1 k_1' K_1 If a circle is set and you need to construct a straight line, which is an image of the circle, then this is easy to do: we construct a reverse with points, through it, perpendicular to the straight line. A A' OA

K_1 If a circle intersects a reversal circle, as follows K_1

MN There will be a straight line passing through the intersection of the given circle and the inversion circle. M, N K_1

4. A straight line diagram that does not go through the inversion center is a circle that passes through the inversion center.

5. When reflecting the image, the circle that does not pass through the center of the inversion will be a circle.

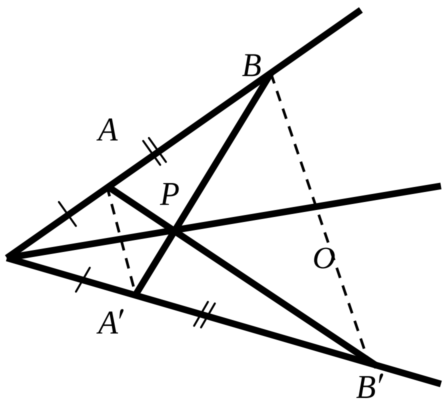
2.2.2 Application of the geometric transformation in the decomposition of the proof problem

(1) Axial symmetry method

Task 1.

On each side, the angle of a vertex is chosen by two points under the condition:. Proprove that the intersection is straight and located on the double branch at a given angle. O A,B A',B' $|OA|=|OA'|,|OB|=|OB'|$ P AB' $A'B$

evidence:



perpendicular to the two cuts and divides them into half. Therefore, in the axisymmetric case, the axis is bipartitioned and is a straight direct correspondence. So, and intersect on one axis-a double partition of one angle. AB' $A'B$ AB' $A'B$

AA',BB' Obviously, the double partition of this angle is

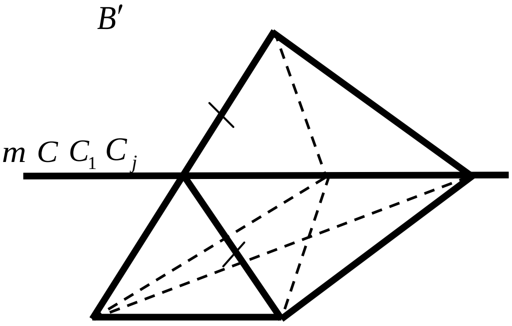
Task 2.

Demonving that the isosceles triangles have the smallest circumference among all isoscelic triangles with common bases.

evidence:

B'

m C C_1 C_j



A B

The heights of all isosceles direct, parallel.(Such a direct triangles with common bases are two. They are equal to each other equal to each other. Thus, the AB C m AB third vertex of each of them is

AB Lie down from the straight place and from her different sides).

B' B We will construct the image of the point under axisymmetric axisymmetry. For each point, we have to go directly to:. Tom. When the intersection with the straight line coincides, we arrive. m C_j m

$|BC_j| = |B'C_j|$ $|AC_j| + |BC_j| > |AB'|$ C_j C AB' m $|AC| + |BC| = |AB'|$

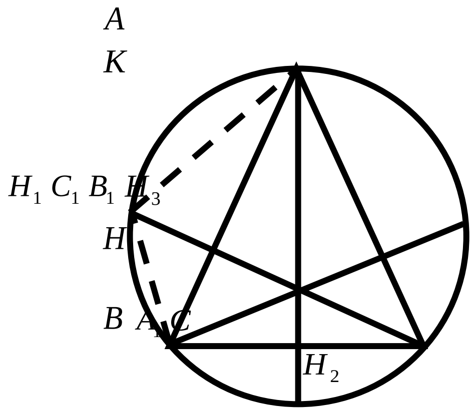
ΔABC That way, he would be equal. Indeed, only when the point coincides with the point, in our triangle. C_j C $\angle A = \angle B$

Thus, among all isoscelic triangles with a common base, the smallest circumference has an isoscelic triangle.

Task 3.

ΔABC H H The intersection point with its height is given. Proprove that the point of symmetry relative to the edge of this triangle is located on the circle described around it.

evidence:



K Let- -the circle described around the triangle, - -the height of the triangle, - -their intersection. ABC AA_1, BB_1, CC_1 H

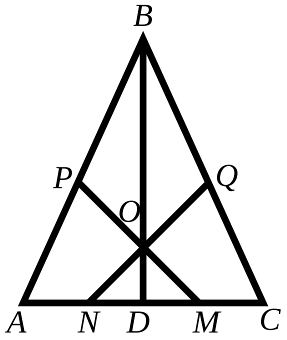
H_1 Consider the points corresponding to the PRI points. then. H $S_{(AB)}$ $\angle AHB = \angle AH_1B$

perhaps,. $\angle AHB = \angle A_1HB_1$, $\angle A_1HB_1 + \angle A_1CB_1 = 180^\circ$ $\angle HA_1C +$ $\angle CB_1H = 180^\circ$ $\angle AH_1B + \angle BCA = 180^\circ$ K In this way, and the circle through H_1 Similarly, the proof point, relatively symmetric, is located on the circle. H_2 H_3 H BC, CA K

Task 4.

Delayed cutting level based on the isosceles triangle AVS In AM and CN, the lateral incision plane AP and CQ. Proof cutting On the height BD of this triangle, the PM and the QN intersect.

evidence:



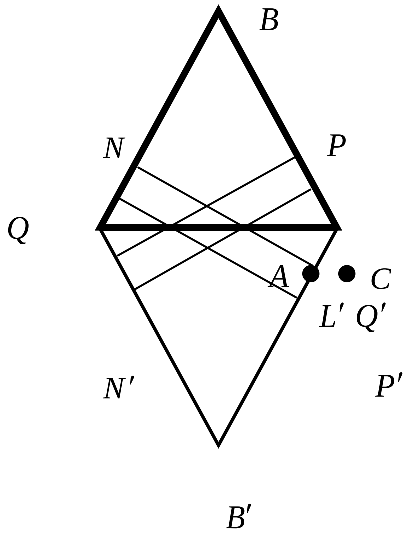
considering the axial symmetry, its axis will be a straight line BD. Given the nature of the isosceles triangle, point A is shifted up in C, point B-B, and section AB-shifted up in CB. cut

Let the cutting data delay as shown in the figure, AM is equal to CN, AP and CQ, so according to the axisymmetric properties, point N will move to point M and P at point Q. Therefore, NQ section is symmetric relative to the BD axis of MP section, so NQ and MP sections (image and image) will pass through the BD axis at the same O point, which belongs to height BD.

objective 5.

We prove that in an isosceles triangle, the sum of the distance of each base point from the edge is a constant.

evidence:



$L\ K\ M$

ABC $AB = BC$ K, M AC Let the triangle be isosceles (), the dot belongs to the bottom of the triangle. We prove that: -Great is steel. ΔABC $LK + KP = NM + MQ$

Consider the axial symmetry of the triangle relative to its base. ABC AC

The result in a diamond. The line will pass through accordingly. Because, there is the height of the diamond, and the height is the size of the steel. Similarly, the proof $ABCB'$ $LK, NM,$ PK, QM $L'K, N'M, P'K, Q'M$ $KP = KP'$ $LP' = LK + KP$ LP'

$NM + MQ$-Has become a great: because, there-$MQ = MQ'$ $NQ' = NM + MQ$ NQ' The height of the diamond, the height is the size of the steel.

(2) Same-state method

Task 1.

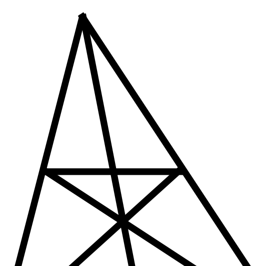
We prove that in any trapezoid, the intersection of the edge extension, the diagonal intersection, and the base center are in a straight line.

evidence:

Let ABCD be an arbitrary trapezoid, M, P, N, and L expressed in the four-point condition. Consider the homomorphisms, where. $H_M^{k_1}$

$k_1 = \dfrac{AD}{BC}$

M



B N C

P

A O L

middle of the homomorphic incision). Thus the points M, N, L are direct.

Now consider the homomorphism, in the. $H_M^{k_2}$ $k_2 = -\frac{AD}{BC}$

$H_M^{k_1}(B) = A, H_M^{k_1}(C) = D$ $H_M^{k_1}(BC) = AD,$ $H_M^{k_1}(N) = L$ Yes:, and then (as the

$H_P^{k_2}(B) = D, H_P^{k_2}(C) = A$ $H_P^{k_2}(BC) = DA$ $H_P^{k_2}(N) = L$ And get it, and then, ah. This means that the P, N, and L points belong to the straight line. Derving from the considered homomorphisms, points M, P, N, L belong to a straight line.

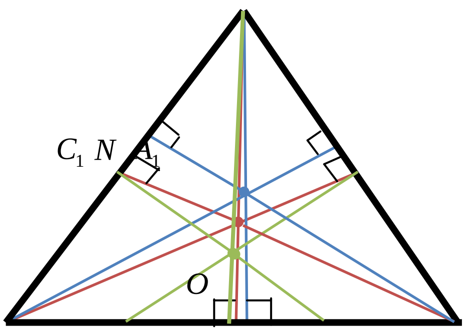
Task 2.

We prove that for any triangle ABC, the height intersection point N point, the median intersection M point and O point are the center of the described circle in a straight line, and M is between O and N, divided by the proportional section of O. $\lambda = \frac{1}{2}$

evidence:

$H_M^{\frac{1}{2}}$ $H_M^{\frac{1}{2}}(A) = A_1, H_M^{\frac{1}{2}}(B) = B_1, H_M^{\frac{1}{2}}(C) = C_1$ A_1, B_1, C_1 Consider homomorphism. So, where is the middle of the triangle edge, respectively.

B



M

A B_1 C

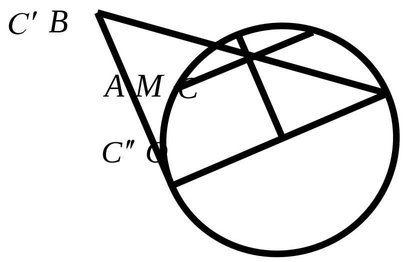
A line containing the triangle height, oriented perpendicular to the edge. Yes, especially directly, because. Since the homology point and center are point, where M $CN \to C_1O$ $H_M^{\frac{1}{2}}(N)=O$ collinear, N, O, M are a straight

Belongs to incisions and. $ON\ OM:MN=1:2$

Task 3.

Three points A, B, and C on the circle are given and the symmetry points of points C ′ and C ′ relative to the section of A and the center of the circle, respectively, to prove the sections S ′ and C ′ of A ′.⊥

evidence:



Consider homomorphism:. From here. H_C^2 $H_C^2(M)=$ C', $H_C^2(O)=C''$, $H_C^2(OM)=C'C''$

$Z_M(C)=C'$ $Z_O(C)=C''$ Go in and what to have, aSS ″ =2SO. $CC'=2CM$

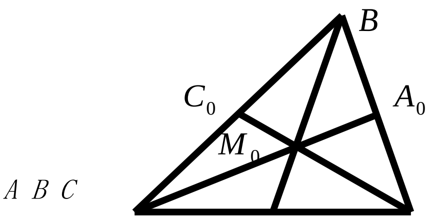
What to get. $C''C' \| OM$

⊥Because OMAV (OM belongs to the radius of the circle through the middle of the string), so $C''C' \perp AB$.

Task 4.

It prove that the median of each triangle at its intersection is divided by a 2:1 ratio, calculated from the vertex.

evidence: ABC M_0 Let a given triangle, a median intersection. Consider the central homomorphism of the



triangle, because of. AA_0, BB_0, CC_0 Γ_0 ABC $A_0 = \tilde{A}_0(\grave{A}), \hat{A}_0 =$ $\tilde{A}_0(\hat{A}), \tilde{N}_0 =$

$= \tilde{A}_0(\tilde{N}), \grave{I}_0 = \tilde{A}_0(\grave{I}), \grave{A}_0\grave{I}_0 = \frac{1}{2}\grave{A}\grave{I}_0, \hat{A}_0\grave{I}_0 = \frac{1}{2}\hat{A}\grave{I}_0, \tilde{N}_0\grave{I}_0 = \frac{1}{2}\tilde{N}\grave{I}_0$.

objective 5.

ΔABC *Proprove that a point symmetric with respect to its edge center belongs to that described around this circular triangle.*

evidence:

H $k = 2$ When homing with the coefficient of the center and the circle in the point (the triangle height), the nine points turn to the circle described around the triangle, and the point (the center of the edge), for example at the point, the symmetrical point. Because a point belongs to a circle of nine points, so ABC M_1 BC H M_1 I prove the theorem of the task.

Task 6.

We prove that for any point belonging to the edge of a double-sector triangle, the sum of the distance from the corresponding edge (or difference mode) is equal to the sum of the distance from the third edge.

evidence:

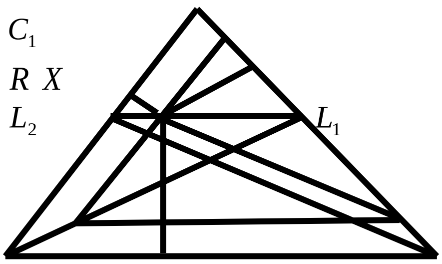
$L_1L_2L_3$ L_1, L_2, L_3 Triangles are called bisectors, if the dots are bisector bases omitted according to the sides of the triangle. CB, AC, AB ABC

C

C_1

R X

L_2



L_1

A_1 Q B_1

A L B

X L_1L_2 P, R, L Let the point belong to the incision. Let- -the projection of this point on the triangle edge. Consider the central and coefficient homomorphisms. ABC L_1 $k = \frac{L_1X}{L_1L_2}$

$A_1B_1C_1$ ABC BL_2 B_1X $A_1B_1C_1$ X A_1B_1, B_1C_1 $XQ = XP$ A_1 AL_1 A_1B_1, A_1C_1 Then the triangle will be a homomorphic triangle, and the image of the double partition is the double partition of the triangle. So the distance to the edge is equal, that is. Since the points belong to double partitions, they are directly equidistant by the straight line. So, and then, or. AB, AC $QL = XR$ $XQ + LQ = XP + XR$ $XL = XP + XR$

X L_1L_2 If a point belongs to the extension of a cut, then the decomposition is similar, but will need to consider the difference mode of the corresponding distance instead of the sum of the distance.

(3) Rotation method

Task 1.

C *Two parallel lines and an unequal point are given, proving that such points will be found on the line and the lines, respectively, that the triangle will be equilateral.* a b A, B ABC

evidence:

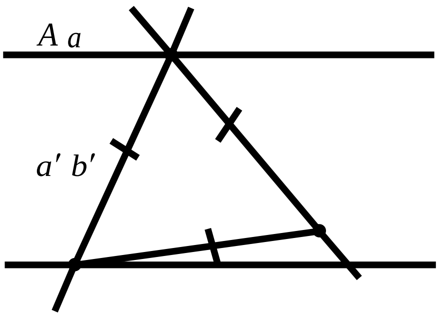
C $\alpha = -60^{\circ}$ Consider a rotation with center and rotation angles (clockwise motion) and find a straight image. The intersection of the line lets us through. If now a' a a' b B

A a

C b

B

a' b'



Considering a backward first transformation. Would be transformation, we will directly parallel, because. a A B ΔABC find the image of the point in the $CB = CA, \angle BAC = 60^{\circ}$

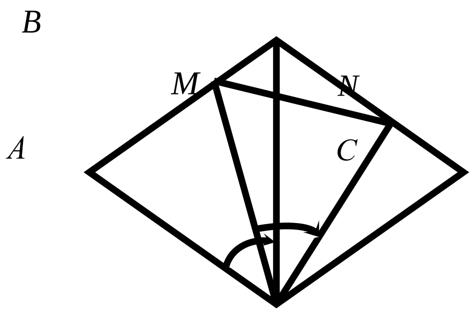
Task 2.

AB, BC $ABCD$ $\angle BAD = 60^{\circ}$ *On the sides of the diamond, the dots are marked in such a way. Proved to be correct.* M, N $AM = BN$ ΔMDN

evidence:

Easy to determine and. Tom $AD = DB = DC$ $\angle ADB = \angle BDC = 60^{\circ}$

$$\left.\begin{matrix} R_D^{-60^{\circ}}(A) = B \\ R_D^{-60^{\circ}}(B) = C \end{matrix}\right| \Rightarrow R_D^{-60^{\circ}}(AB) = BC, M \in AB,\ N \in BC, AM = BN \Rightarrow R_D^{-60^{\circ}}(M) = N \Rightarrow$$



$\begin{cases} DM = DN, \\ \angle MDN = 60^{\circ} \end{cases} \Rightarrow \Delta MDN$ −parallel.

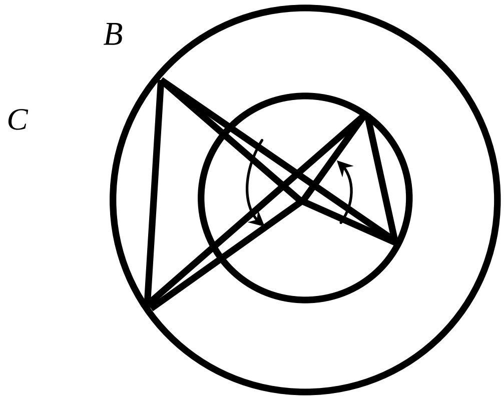
Task 3.

AB, CD *Let two of these strings be in concentric circles, what is the center. What to prove.* ω, ω_1 O $\angle AOB = \angle COD = \alpha$ $AC = BD$

evidence:

When a dot turns around an angle, the point turns to a point, and when it turns around an angle to a point, it turns to a point B O

α B A D O α



O D
A ω_1 ω

C BD drop. So when you go around line. O α AC $BD = AC$ an angle of point, the straight direct,. AC $BD = AC$

Task 4.

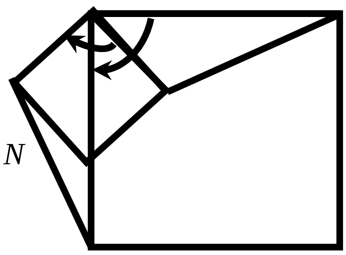
AB, BC ABC $ABMN, BCPQ$ $ABMN$ ABC A square is constructed on both sides of the triangle, and is located on both sides of the square and triangle, and on one side of the square. What to prove. AB $BCPQ$ ABC BC $MQ = AC, MQ \perp AC$

evidence:

A B $\alpha = -90^\circ$ M C When the turning point of a point around an angle becomes a point, when the turning point of a point around an angle becomes a point. So when you go around a corner, go straight. $\alpha = -90^\circ$ Q AC B $\alpha = -90^\circ$ MQ

B C

M A



N

Q P

From here, we have something like:

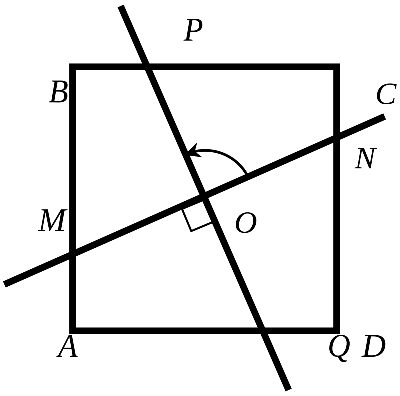
$$\begin{cases} AC = MQ, \\ AC \perp MQ. \end{cases}$$

objective 5.

Through the center of the square, two straight lines perpendicular to each other are conducted. Proprove that the cuts located inside the square are equal.

evidence:

$R_O^{90^\circ}$ We consider the turning around. Move here. Tom. Belongs to the point $A \to D, B \to A, C \to B, D \to C$ $AB \to DA, BC \to AB, CD \to BC, DA \to CD$ M



the intersection of the direct image mentioned above. Because of what. From here, and then to the next. Similarly, Tom.. $MN \to QP$

$\angle MOQ = 90^{\circ}; AB \to DA$

$R_O^{90^{\circ}}(M) = PQ \cap AD = Q$ $R_O^{90^{\circ}}(N) = P$

$R_O^{90^{\circ}}(MN) = QP$

MN, AB At the same time is direct, so that its image can be found as so. $MN = PQ$

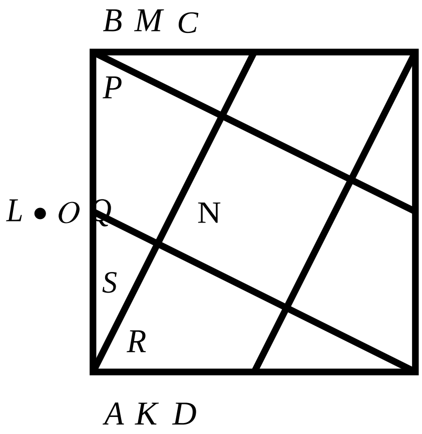
Task 6.

A, B, C, D $ABCD$ *The top of the square connects to the middle of the sides. The proof point is the vertices of the square.* M, N, K, L BC, CD, DA, AB $P, Q, R, S (P = BN \cap AM, Q = BN \cap KC, R = KC \cap DL, S = DL \cap AM)$

evidence:

$AC \cap BD = O$ $R_0^{90^{\circ}}(ABCD) = DABC$ Let, and then. What is inferred from here.

Tom $R_0^{90^{\circ}}(L) = K$, $R_0^{90^{\circ}}(K) = N$, $R_0^{90^{\circ}}(N) = M$, $R_0^{90^{\circ}}(M) = L$ $R_0^{90^{\circ}}(S) = R_0^{90^{\circ}}(AM \cap DL) =$



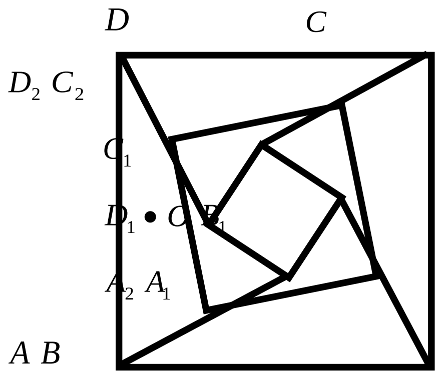
$= R_0^{90^{\circ}}(AM) \cap R_0^{90^{\circ}}(DL) = DL \cap CK = R$ same, .

$R_0^{90^{\circ}}(R) = Q, R_0^{90^{\circ}}(Q) = P, R_0^{90^{\circ}}(P) = S$

$R_0^{90^{\circ}}$ $SRQP$ Therefore, when you rotate, the quadrilateral appears on yourself, so it is a square.

Task 7.

$ABCD$ $A_1B_1C_1D_1$ *There is a square in the middle of the square, and the center of the square coincides. We prove that the middle part of the cut serves as the vertex of the new square.* AA_1, BB_1, CC_1, DD_1



evidence:

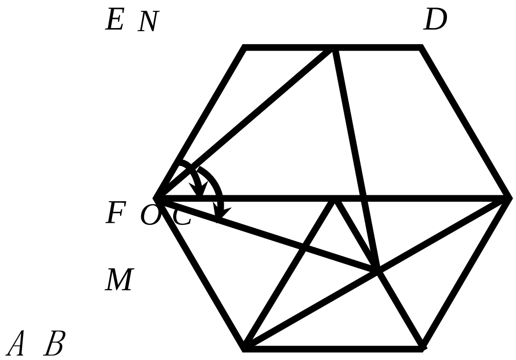
$$\left.\begin{array}{l} R_0^{90^\circ}(AA_1)=BB_1, \\ R_0^{90^\circ}(BB_1)=CC_1, \\ R_0^{90^\circ}(CC_1)=DD_1, \\ R_0^{90^\circ}(DD_1)=AA_1, \end{array}\right| \Rightarrow \left\{\begin{array}{l} R_0^{90^\circ}(A_2)=B_2, \\ R_0^{90^\circ}(B_2)=C_2, \\ R_0^{90^\circ}(C_2)=D_2, \\ R_0^{90^\circ}(D_2)=A_2, \end{array}\right| \Rightarrow$$

$$A_2B_2C_2D_2 == R_0^{90^\circ}(D_2A_2B_2C_2) \Rightarrow$$

$A_2B_2C_2D_2$ −square.

Let. $AC \cap BD = O$

Task 8.



the center of the edge. Proving of what is right. $ABCDEF$ M AC N DE ΔMNF

In a right triangle, the point is the center of the diagonal, and

evidence:

O Let−Data Center hexagon. then− $ABCD$

Lamber and Tom are in between.

Consider turning now M OB $R_F^{-60^\circ}$

And we find the dot image. We have, −for. N $R_F^{-60^\circ}(E)=O$ $R_F^{-60^\circ}(D)=B \Rightarrow$

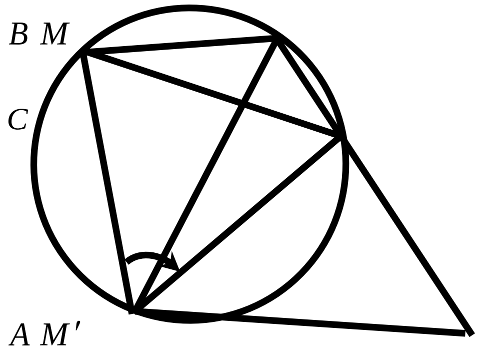
$$R_F^{-60^\circ}(D)=B \Rightarrow R_F^{-60^\circ}(ED)=OB \Rightarrow \qquad R_F^{-60^\circ}(N)=M \Rightarrow \left\{\begin{array}{l} FN=FM, \\ \angle NFM=60^\circ \end{array}\right. \Rightarrow \Delta NMF$$

objective 9.

On the circle described around the straight triangle, take any point. Proprove that the length of the largest intercept is equal to the sum of the lengths of the latter two intercepts. ABC M MA, MB, MC

evidence:

$MA = MB + MC$ Lets prove that.



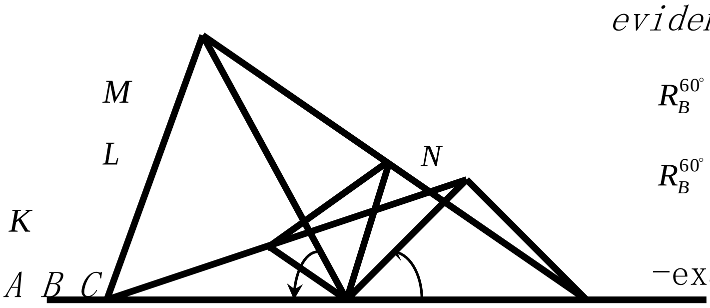
If so, we will consider turning around. Obviously, what. Let, then the equilateral, meaning. $R_A^{-60^\circ}$ $R_A^{-60^\circ}(B) = C$ $R_A^{-60^\circ}(M) = M'$ $\Delta AMM'$ $MA = MM', \angle AMM' = 60^\circ$

$MB = M'C$ $\angle AMC = \angle ABC = 60^\circ$ Otherwise, but (as written), so the integral lies on a straight line. We now find that: M, C, M' $AM = MM' = MC + M'C = MC + MB$.

Task 10.

AB,BC AC *Construct an equilateral triangle on a continuous cut from one side of its line. Jean-middle, middle. Proof of what is an equilateral one.* ABM,BCN K AN L MC ΔKBL

evidence:



$R_B^{60^\circ}(C) = N, R_B^{60^\circ}(M) = \quad A \Rightarrow R_B^{60^\circ}(CM) = NA \Rightarrow$

$R_B^{60^\circ}(L) = K \Rightarrow \begin{cases} BL = BK, \\ \angle LBK = 60^\circ \end{cases} \Bigg| \Rightarrow \Delta KBL$

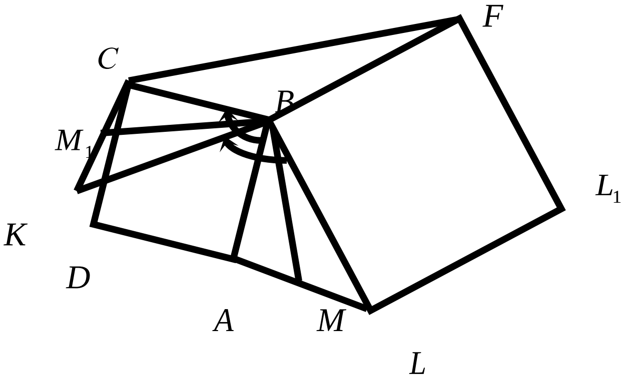
-exactly.

Task 11.

$ABCD, LBFK$ B *Blocks have a common vertex. Demonving that the median of the triangle is perpendicular to.* BM ABL CF

evidence:

$R_B^{-90^\circ}$ $R_B^{-90^\circ}(L)=L_1, R_B^{-90^\circ}(M)=M_1$Consider it, and let go. Understand the point



L_1, B, F B L_1F $R_B^{-90^\circ}(\Delta BAL)=\Delta BCL_1$ M_1 CL_1 ΔL_1CF BM_1 Lie on a straight line, in the middle. Because what, thats in the middle. Now in an incision, midline and, meaning. $BM_1 \| CF$

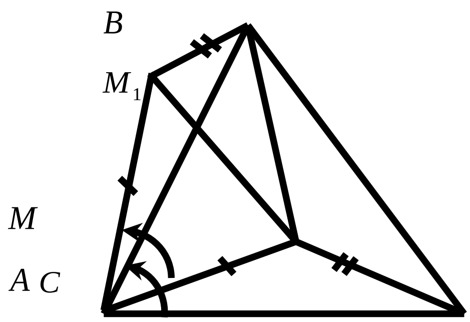
It means that. $BM \perp CF$

Clearly, we have proved that the. $CF = 2BM$

Task 12.

Take any arbitrary point in the middle of the equilateral triangle. It is shown that triangles can be constructed from the cut. ABC M MA, MB, MC

evidence: $R_A^{60^\circ}$ $R_A^{60^\circ}(M)=M_1$ $AM = AM_1 = MM_1$ We consider the turning around. Let it go, and then. It is very easy to find something. It is now quite clear, that what was being searched. $R_A^{60^\circ}(C)=B \Rightarrow MC = M_1B$ ΔMM_1B

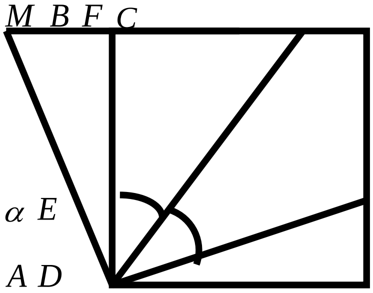


Task 13.

CD $ABCD$ E *A point was built on the side of the square. The bipartitions of the corner intersect at a point. What to prove.* BAE BC F $AE = ED + BF$

evidence:

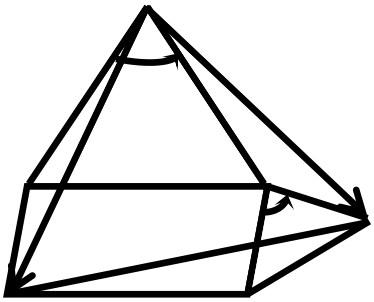
$R_A^{90^\circ}$ Consider the turn, and let this turn be reflected in ΔADE



ΔABM $AE = AM, DE = BM$. then. What will we show you. forsooth,. Now we have:. $AM = MF$ $\angle MAF = \angle BFA = 90^\circ - \alpha$

$AE = AM = MF = MB + BF = BF + DE$

target 14.



outside, are equilateral. Proof of what is an equilateral one. $\Delta ABM, \Delta BCN$ ΔDMN

evidence:

If we show, this task will be fired. $R^{60^\circ}\left(\overline{ND}\right) = \overline{NM}$

AB, BC $ABCD$ *The two adjacent edges of the parallelogram, from the*

$R^{60^\circ}\left(\overline{ND}\right) = R^{60^\circ}\left(\overline{NC} + \overline{CD}\right) =$ $R^{60^\circ}\left(\overline{NC} + \overline{BA}\right) =$ $= R^{60^\circ}\left(\overline{NC}\right) + R^{60^\circ}\left(\overline{BA}\right) =$ $\overline{NB} + \overline{BM} = \overline{NM}$. In fact, this is an equal. $R^{60^\circ}\left(\overline{ND}\right) = \overline{NM} \Rightarrow \Delta DNM$

Task 15.

Parallel ogram, from the outside, square construction. Proprove that the center of these squares is their vertices.

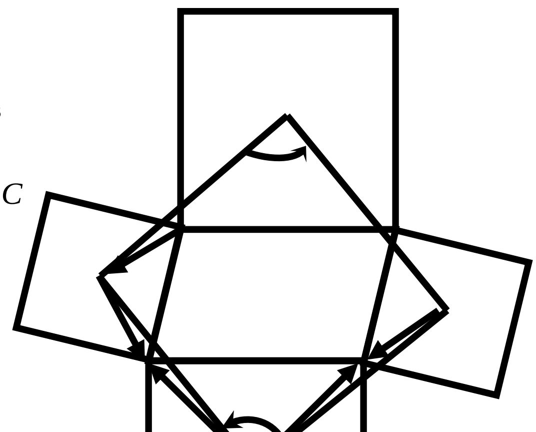
evidence: O_4

O_2

A

D

O_1



Enough to show us that two $\overline{O_1O_4} = R^{90^\circ}\left(\overline{O_1O_2}\right)$ $\overline{O_3O_2} = R^{90^\circ}\left(\overline{O_3O_4}\right)$ equals are fair: and. To prove our equality. We have:. Tom

$$\overline{O_1O_2} = \overline{O_1D} - \overline{O_2D} = \overline{O_1D} - \overline{BO_4}$$

$$R^{90^\circ}\left(\overline{O_1O_2}\right) = R^{90^\circ}\left(\overline{O_1D} - \overline{BO_4}\right) =$$

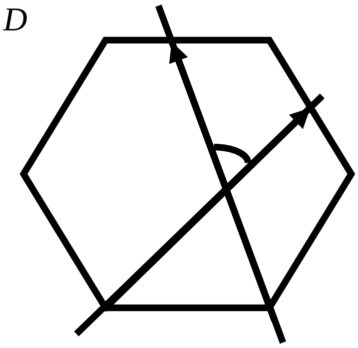
$$= R^{90^\circ}\left(\overline{O_1D}\right) - R^{90^\circ}\left(\overline{BO_4}\right) =$$

$= \overline{O_1A} - \overline{O_4A} = \overline{O_1O_4}$. Similarly, the second equality is also proved.

Task 16.

In the correct hexagon, through the corresponding $ABCDEF$ M, N CD, DE *On both sides of the middle. At which angle does the line intersect?* AM, BN

evidence: The pattern analysis yielded the assumption of the equal magnitude of the search angle. And really, 60°



$$\overline{AM} = \overline{AB} + \overline{BC} + \frac{1}{2}\overline{CD} =$$

$$= \overline{AB} + R^{60^\circ}\left(\overline{AB}\right) + \frac{1}{2}R^{120^\circ}\left(\overline{AB}\right).$$

Tom $R^{60^\circ}\left(\overline{AM}\right) = R^{60^\circ}\left(\overline{AB}\right) + R^{120^\circ}\left(\overline{AB}\right) +$

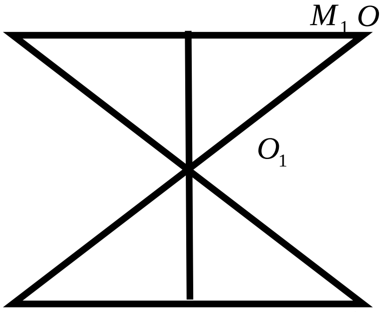
$+\frac{1}{2}R^{180^\circ}\left(\overline{AB}\right) =$ $\overline{BC} + \overline{CD} + \frac{1}{2}\overline{DE} = .\overline{BN}$

(4) Centric symmetry method

Task 1.

O O_1 Φ O_2 O O_1 Φ *Let the point and the symmetrical center of- - Fig. Demonprove that the relatively symmetrical points will also be the symmetry center of the graph. Can we infer the number of symmetric centers of a finite graph from here?*

evidence: points, and relatively symmetrical



points. Next, consider the point, the symmetry M Φ M_1 M O_1 M_0 M_1 O M_2 M_0

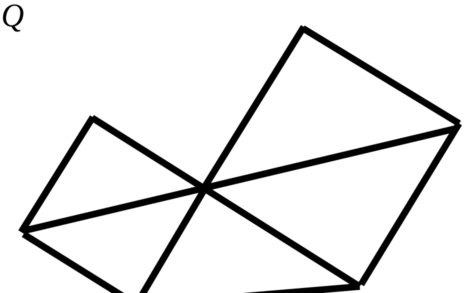
opposite. All four points, which belong to the. Or points O_1 M M_0 M_1 M_2 Φ

Take any point in the graph, consider relatively symmetrical M M_2 O_2 O_2 Φ Φ And it is relatively symmetrical. From follows, that it is the center of symmetry. Obviously, each bounded graph can have only a one symmetric center, because similarly, it can be shown that relatively symmetric points would also be symmetric centers of the graph, etc. O_3 O_1 O_2 Φ

Task 2.

AC, CB $ACMN, BCQP$ PN *There are squares on both sides of the triangle. We prove that the straight line always passes through the same point between the fixed point position and the fixed point position.* A, B C

evidence: M P



N B

A

B 90° P C The point image does so when crossing the center and corner, and it does when crossing the center and corner. So the composition of these turning points is translated A 90° C N P

N At the point. But the composition of the specified turn is the central symmetry with the specified center. Therefore, the straight lines connecting the corresponding points will always pass through. O

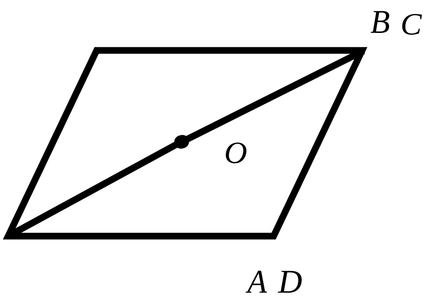
P, N O

Task 3.

In the quadrilateral form:. To prove what a parallelogram is.

$ABCD$ $AB \| CD, AB = DC$ $ABCD$

evidence: $BC \| AD$ AC O AB, CD We need to prove that the. To this end, we will take a diagonal line and find it in the middle. The beam is parallel in the opposite direction (because it is located in



AC Different half-planes with boundaries). Then, when relatively centrosymmetric, the beam will be reflected onto the beam. In doing so (because of what O AB CD $A \to C$

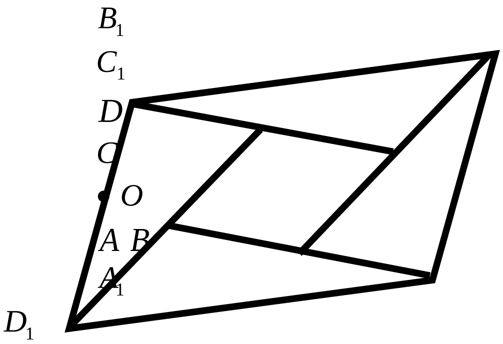
$O \in AC, OA = OC$ $B \to D$ $AB = CD$), And (because of). The rays is derived from here

BC, AD Centrally symmetric, and so. $BC \| AD$

Task 4.

$ABCD$ A_1, B_1, C_1, D_1 *Let a parallelogram, the point on the extension of it, which is an average, C is an average, and an average in the middle. The proof is a parallelogram.* B AA_1 BB_1 D CC_1 A DD_1 $A_1B_1C_1D_1$

evidence:



$$\left.\begin{array}{l} Z_0(A)=C, \\ Z_0(B)=D, \\ Z_0(AB)=CD, \\ AA_1=CC_1, \end{array}\right| \Rightarrow Z_0(A_1)=C_1, Z_0(C_1)=A_1.$$

same, . $\begin{array}{l} Z_0(D_1)=B_1, \\ Z_0(B_1)=D_1. \end{array}$

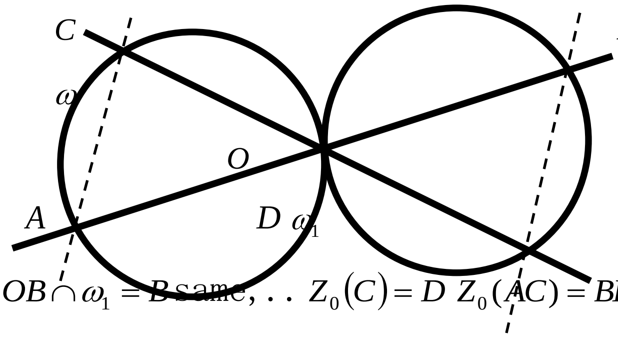
O So, the symmetry center

$A_1B_1C_1D_1$ $A_1B_1C_1D_1$ And Tom is a parallelogram.

objective 5.

ω, ω_1 O *Two grades of circles touch on the dots. Two straight lines across a point at the point and. What to prove.* O A, B C, D $AC \| BD$

evidence:

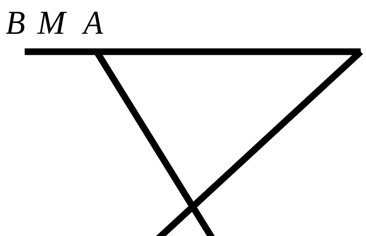


$Z_0(A)=Z_0(OA\cap\omega)=OB\cap\omega_1=B$ same, . . $Z_0(C)=D$ $Z_0(AC)=BD \Rightarrow AC \| BD$

Task 6.

We prove that in the centrosymmetric case, each beam in the plane maps to an opposite directional beam.

evidence:



O $O \in AB$ AB $A'B'$ If it and belongs to a direct and obvious.

A' M' B' $AB \uparrow\downarrow A'B'$

$O \notin AB$ $AB \| A'B'$ $AB \neq A'B'$ if. but.

Let. $Z_0(AB) = A'B'$ Lets just pick it out casually. then. $M \in AB$

$Z_0(M) = M' \in A'B'$

M M' Because and lying on a different side, then. AA' $AB \uparrow\downarrow A'B'$

Task 7.

We prove that the displacement of each beam in the plane mapping to the beam in the opposite direction is centrosymmetric.

evidence:

f Let- -meet the conditions of the move. Any consideration. Let. then. Next we have and, yes. Let-in the middle. Lets think about it. Using the previous task and equality and,,. Then thats it. Since only one displacement maps to, then. ΔABC $f(\Delta ABC) = \Delta A'B'C'$

$AB = A'B', AC = A'C', BC = B'C'$ $f(AB) = A'B'$ $f(AC) = A'C'$ $AB \uparrow\downarrow A'B'$ $AC \uparrow\downarrow A'C'$ O AA' Z_0 $AB = A'B', AC = A'C', BC = B'C'$ $AB \uparrow\downarrow A'B'$ $AC \uparrow\downarrow A'C'$ $Z_0(A) = A', Z_0(AB) = A'B', Z_0(B) = B', Z_0(AC) = A'C', Z_0(C) = C'.$ $Z_0(\Delta ABC) = \Delta A'B'C'$ ΔABC $\Delta A'B'C'$ $f = Z_0$

Task 8.

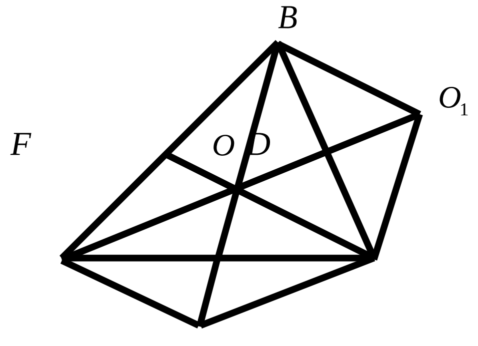
Proving that the median value of a triangle intersects at a point.

evidence:

A E

C

O_2



AD, BE ΔABC O CO Let-median and-their intersection points. Let us proceed directly, considering centrosymmetry. It will have an image, which together with its shape forms a parallelogram D ΔCOB ΔBO_1C

CO_1BO ΔCOA E $AOCO_2$ $CO_1 \| OB, CO_1 \| O_2O, O_2C \| OA, O_2C \| OO_1$ CO_2OO_1 $CO_1 = OO_2$ $CO_1 = OB$ $OO_2 = OB$ $CO \| AO_2$ CO AB F AB CF O $OE = EO_2$ $OB = OO_2$ $BO : OE = 2:1$. Similarly, in an image with a central symmetry, we obtain a parallelogram. From here we have:. Thus, the quadrilateral is a parallelogram and. Given the ratio, we will get:.in addition. So intersect at a point, which is a middle. Thus, the third median passes through a point. Considering, for example, and, it is proved that. Similarly, we show that the median values of the other intersections are also split in a 2:1 ratio.

(5) Parallel transfer method

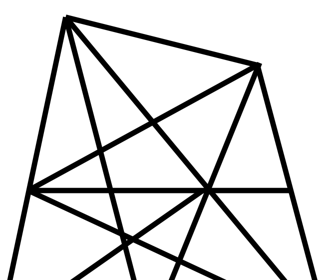
Task 1.

We prove that when the area of an ABC triangle is marked by S, the edge of the triangle constitutes S as the area of the median value of the triangle. $\frac{3}{4}$

evidence:

Let AA$_1$、BB_1、CC_1 ΔABC -median. Well build it.obvious, $B' = T_{\overline{AB_1}}(AB)$ $C_1B' = T_{\overline{AC_1}}(AA_1)$ $\Delta C_1B`C$ While both sides will be the median =S, because =. $\Delta ABC$ $S_{\Delta CA_1B`}$ $\frac{1}{4}$ $S_{\Delta A_1B`C}$

B
B'



P

C_1 A_1 N

M
K
A C

B_1

$\frac{1}{4}\cdot\frac{1}{2}ch_c =$ $\frac{1}{4}S_{\Delta ABC}(CK=\frac{1}{2}h_c)$ Where K-the base height from the top of C to B ′ B_1. It is not difficult to determine point A_1There are median intersections, B, C_1N,CP delta$_1B$.

$S_{\Delta A_1B_1C} = \frac{1}{2}\cdot A_1B_1\cdot CK = \frac{1}{2}\cdot\frac{1}{2}c\cdot\frac{1}{2}h_c =$

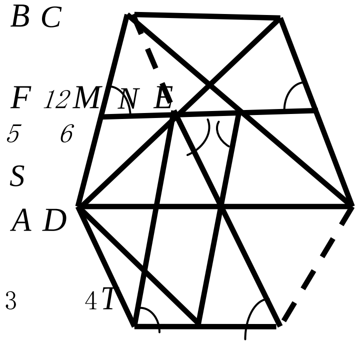
Because in the triangle of C_1B ` M and CB ` M baseline C_1 $S_{\Delta C_1B`M}$ $S_{\Delta CB`M}$ $S_{\Delta C_1A_1M}$ $S_{\Delta CA_1M}$ M and CM and the common height, and then =. so =. minus the initial last equation, we get = or =. $S_{\Delta C_1B`M}$ $S_{\Delta C_1A_1M}$ $S_{\Delta CB`M}$ $S_{\Delta CA_1M}$ $S_{\Delta C_1B`A_1}$ $S_{\Delta CB`A_1}$

Again, we ensure that the =. So the CA triangle is shaped $S_{\Delta CB`A_1}$ $S_{\Delta CA_1C_1}$ $_1B$, $C_1B$, that$_1C_1$, Have the same area, hence =3S=S, which also needs to be demonstrated. $S_{\Delta CC_1B`_1}$ $3S_{\Delta CB`A_1}$ $\frac{1}{4}$ $\frac{3}{4}$

Task 2.

In the rectangle of ABCD, the relative edges of AB and CD are equal. We prove that the line connecting the center of the quadrular diagonal intersects parallel with the edges under equal angles.

evidence: P Q



. Let points M and N be the center of the diagonal lines of AS and BD. The extension section MN intersects the AB surface and CD

surface at point F, we perform the equal and parallel MR, and CD parallel shift of VA surface and surface is equal and parallel MQ. CD surface:,, then VA surface is $MP = T_{\overline{BM}}(BA)$ $MQ = T_{\overline{CM}}(CD)$

$\angle 3 = \angle 4$ $\angle 5 = \angle 6 = \angle 4$ $\angle 2 = \angle 5 = \angle 4$ $\angle 6 = \angle 1$However, in the condition of BA=CD, the triangle MPQ is isosceles (MP = MQ), therefore, MS is the center line of the ACD triangle, therefore, S point is the center line of the AD edge, from here, NS is the center line of the ABD triangle. The triangular MSN and STQ are isosceles and therefore. The quadrads of ABNT, MCDQ and PMNT are parallelograms, starting here (multifaceted in parallel lines). So, what she needs to prove. $\angle 1 = \angle 2$

Task 3.

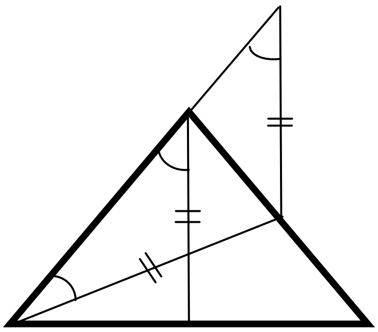
Proving that two triangles with median congruents will be isosceles.

evidence:

B'

B

$E(F')$



A F C

ΔABC Consider the same median. We find the image of the median number. Triangles are isosceles because of the. But, Tom.. AE, BF

$F'B'$ FB $AB'F'$ $|AE| = |FB| = |F'B'|$

$\angle FBA = \angle F'B'B$

$\Delta FBA = \Delta AEB, \angle CAB = \angle ABC$

Task 4.

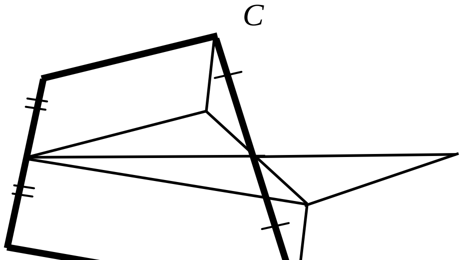
M, N BC, DA $ABCD$ $2|MN| = |AB| + |CD|$ *Middle of the relative edge of the let-quadrilateral. Then, the quadrilateral is a trapezoid.*

evidence: D

M'

M N N''

C



M''
A
B

$ABCD$ Suppose the quadrilateral is not a trapezoid. Consider the parallel transfers, and find their images accordingly. We will achieve parallel cuts between each other. obvious,.

$\overline{DC}, \overline{AB}$ MD, AM CM', BM''
$\angle NBM'' = \angle NCM', BM'' = CM'$

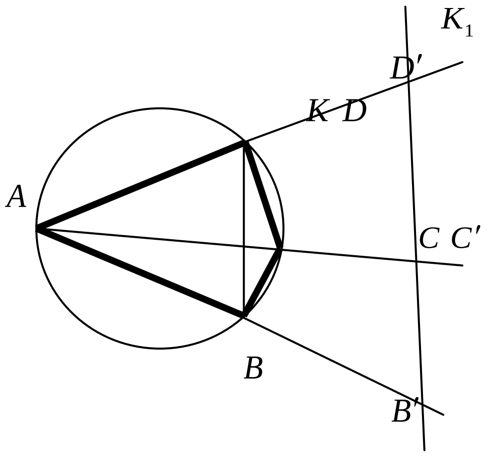
$\Delta NBM'' = \Delta NM'C, \angle BM'' = \angle CNM'\ M'', N, M'\ MM''M'\ MN$ Tom. So the dot is in a straight line, and the graph is a triangle, and its its median value. We conclude from here that there is an inequality:, contrary to the conditions. So the quadrilateral would be a trapezoid. $2|MN| < |MM'| + |MM''| = |DC| + |AB|\ |MN| = |NN''|, |MM'| = |M''N''|\ ABCD$

(6) Inversion method

Question 1. (Ptolemys theorem)

In any circular convex quadragon, the gain of the diagonal is equal to the sum of the gain of the diagonal.

evidence: $ABCD\ K\ D,B\ AC\ A$ Let a convex terelateral whose vertex is embedded into a circle from different sides. We take the vertex as the center of the inversion, with the circle of any radius behind the circle of the inversion. The dot image in this inversion will be the dot lying on a straight line $r\ B,C,D\ B',C',D'$



$K_1\ K$, What is the image of a circle. With inversion sets and attributes, you can write to: $|B'C'| = \frac{r^2|BC|}{|AB||AC|}, |C'D'| = \frac{r^2|CD|}{|AC||AD|}, |B'D'| = \frac{r^2|BD|}{|AB||AD|}$.

Because of what $|B'C'| + |C'D'| = |B'D'|$, $\frac{|BC|}{|AB||AC|} + \frac{|CD|}{|AC||AD|} = \frac{|BD|}{|AB||AD|}$.

From here to the end $|AC||BD| = |AD||BC| + |AB||CD|$.

Task 2.

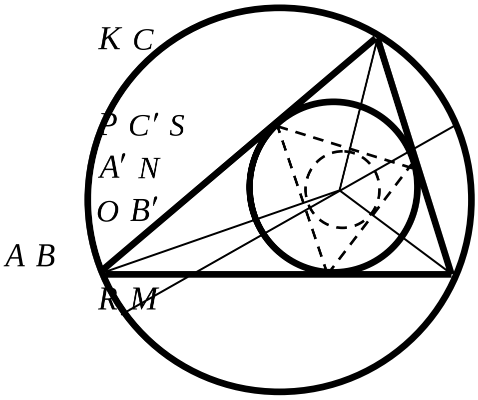
R K *Prove if-describes the radius of the circle around a triangle*

ABC r K_1 *And A is the radius of the circle written in this triangle, then the distance between them*

O, O_1 *The centers of these circles are represented by Eq* $|OO_1|^2 = R^2 - 2Rr.$

evidence:

K_1 For a reversal circle, we take a circle of a triangle by marking its contact point with the edge of the triangle. Then the dot image will be the point, dividing the edges into half (because yes M, N, P A, B, C A', B', C' ΔMNP



the middle of the sides. So this is a circle, and it will be the image of a circle K. The circle has an equal radius. Circle and will correspond to the homomorphism time K_2 K_2 $\frac{1}{2}r$ K K_2

ΔMNP Construct the reverse point).

But his nine circle passed through

O_1 $\frac{1}{2}r : R$ Center on the points and on the coefficients. Now, using the relationship of distance dominance between the two data and the inversion points, we reach an equal conclusion:

$$R : \frac{1}{2}r = \frac{(R + |OO_1|)(R - |OO_1|)}{r^2} \Rightarrow \quad |OO_1|^2 = R^2 - 2Rr.$$

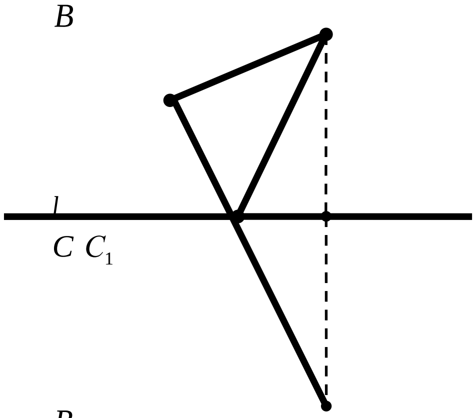
2.2.3 Application of geometric transformation in solving construction problems

(1) Axial symmetry method

Task 1.

The line of l and two points of A and B, which are given on one side of l. Finding such a point on the line l of C, to minimize the circumference of the triangular ABC.

amusement:



point changes, then the circumference of the ABC triangle will also change. But in doing so, the length of the AB side does not change. So on the straight l, you need to find the point C, so that the sum is minimal. $|AC|+|CB|$

C_1 C_1 We take an arbitrary point. If the position of the

B B_1 $|BC|=|B_1C|$ $|AC|+|CB_1|$ Map the points to the relatively direct l.then. Thus, on the line l, require such point C to minimize the sum, that is, between points A and B, so the Raman of ASV has a minimum length, it will be A truncation. AB_1

In fact, a straight incision is shorter than any lamina connecting the end of the incision.

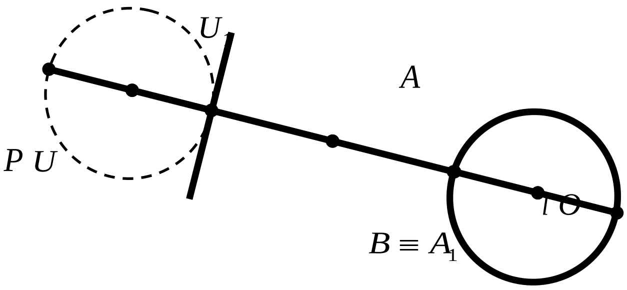
B_1 So we construct a point, that is relatively straight l (axisymmetric). We find the point C (). Point C is a search. The ABS triangle has the smallest circumference. $AB_1 \cap l$

3.

Pass directly through a given point R so that its intersection between a given

U The line l and the given circle are crossed by this point.

amusement:



Assuming that the problem is solved, AB is a direct search. Then Ar = RV, namely the point R is the center of symmetry. So, if you rotate 180° around point R, then Spapard$U$ A_1

With point B, so point B can be found like this.

U_1 Construct a circle, a symmetric circle relative to the point R. U

2. A=。$U_1 \cap l$

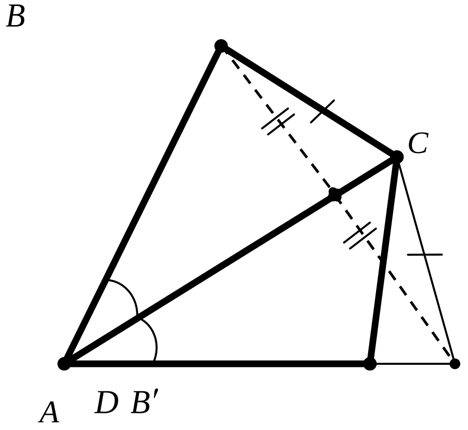
3. We conduct a direct AR, and the AV is a direct search. $AP \cap U = B$

3.

$ABCD$ *Construct a four-sided quadrilateral, if its diagonal is divided by half the angle.* AC A

amusement:

Suppose the quadrof of search are being constructed. We then find a point of relatively diagonal symmetry. Lets get a triangle with all the known edges: (hypothesis). If we delay a cut-off line (which is identical to the edges of the searched quadrangle) and find a point, the symmetric point B' B AC $CB'D$ $|B'C| = |BC|, |B'D| = |AB| - |AD|$ $|AB| > |AD|$ DA DA B B'



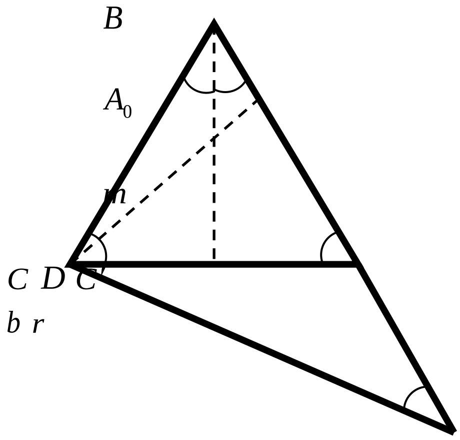
AC Relatively speaking, we find the last two vertices and the looking quadrtet. A B

$CB'D$ The problem makes sense if the tangent from the edge of a triangle meets a condition of the condition: the largest must be less than the sum of the other two.

3.

Construct a triangle, an adjacent angle, and the difference between the other two edges. $|AC| = b$ $\angle A = \alpha$ $|AB| - |BC| = r$

amusement:



α

A

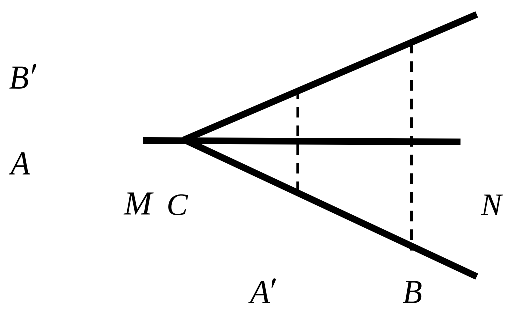
ABC Let the triangle search. Lets take the dichotomy of the triangle as an axis and find a symmetrical point. Points should Arrived with the three known be on the side because the elements BD B m C C' C' AB AB BC straight line will be the image. $\Delta ACC'$ $|AC| = b,$ $|AC'| = |AB| - |BC| = r,$ $\angle A = \alpha.$

$\Delta AC'C$ B If we construct the third vertex of this search triangle, it is not difficult to find: through the middle of a tangent line, we will be perpendicular to it and extend it to the intersection of a straight line at a point. D CC' DB AC' B $|AB| > |BC|$ $\angle A < \angle C$ A $AC'C$ By the way, so the angle is sharp and the angle is blunt. Therefore, in the presence of the problem solution, an inequality should be added, where the baseline is omitted perpendicular to Z as a straight line. $r < |AA_0|$ A_0 C AC'

objective 5.

A B Through the point data and two beams so that the angles between them are directly divided by the data. MN

amusement:



Assuming a ray and satisfying the task conditions. If you take a straight symmetry axis, the beam becomes a beam on the selected plane transformation. CA CB MN CA CB

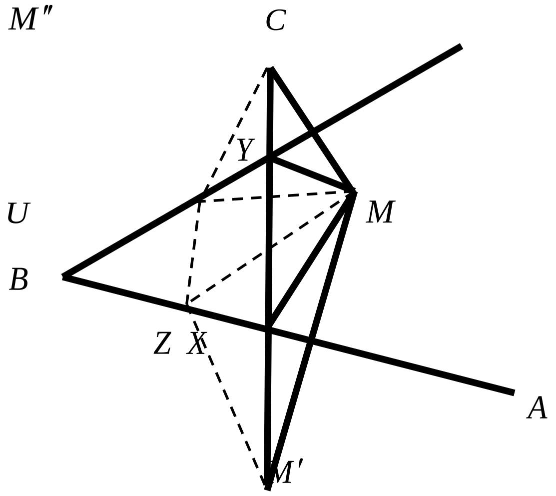
A' A CB B' B CA Therefore, the point as a point image must be located on the beam and the point image on the beam. Such task entertainment comes from here. We construct point I, symmetry point and relative direct. Search for rays lying straight and. A' B' A B MN

AB′ A′B

Task 6.

∠ABC *Now, enter the triangle of the smallest circumference so that one of its vertices is within the given corner point.* M

amusement:



relative to edges and angles. Then we take a straight line, at the point, at the point. The triangle will be searched. In fact, so the circumference is equal to the length of the incision: $M'M''$ AB X BC Y XMY

M' M'' M AB BC Construct point I, symmetric given point $|MX| = |M'X|, |MY| = |YM''|$ ΔXMY $M'M''$

$|MX| + |MY| + |XY| = = |M'X| + |M''Y| + |XY| = |M'M''|$.

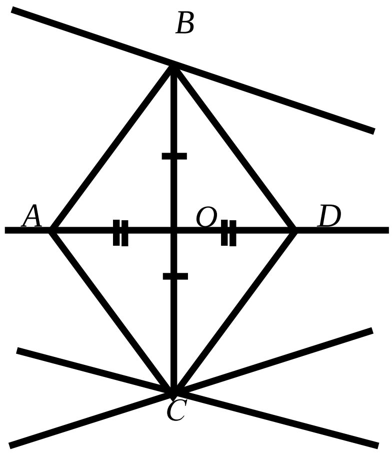
It is easy to determine that the circumference of another arbitrary input triangle will be greater than the circumference (it is equal to the length of a horn). MZU ΔXMY $M'ZUM''$

Task 7.

Build a diamond so that a diagonal is equal to a tangent, its length, and lie on a straight line. While the other two diamond vertices lie on the direct data. r a b, c

amusement:

Let the ABDC- -Romb search, AD=r. We see the construction of Ramba coming down to the construction of one of its vertices, such as C.



relatively direct A symmetry. Thus, in the axisymmetric case, the relatively straight point b enters point C, and the straight b enters some lines passing through point C, so that point C can be constructed as the intersection of the straight C and, one of them b'

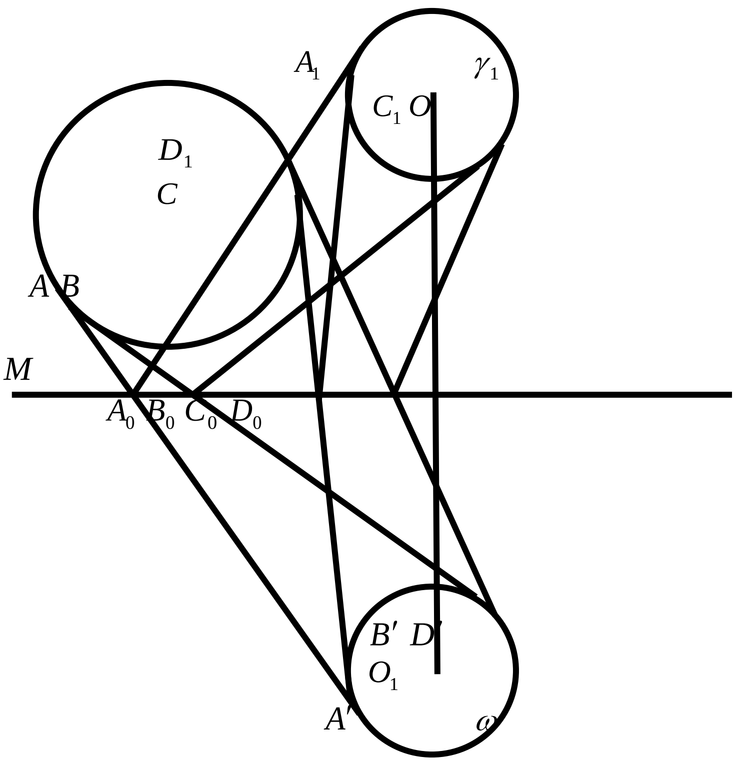
a b' Regarding the properties of Romba points B and C with Give, another is very easy to build.

Task 8.

MN Set straight, and two circles from her side. Finding such a point on this line so that the tangent made from this point to the two circles constitutes the level of the angle of a line. MN

amusement:

γ_2 D B_1
Q.



C'

Marthe two designated circles through and, their centers and respectively. γ_1 γ_2 O Q

ω Lets construct a circle, a relatively symmetric circle. Touch the circle and γ_1 MN γ_2

ω MN Direct through the point of the search,,,,. A_0 B_0 C_0 D_0

AA' Lets show that. The line is an outer cut with the circle sum. Obviously, because the circle and the symmetry are relatively straight, so γ_2 ω $\angle AA_0M = \angle A'A_0N$. γ_1 ω MN $\angle A_1A_0N = \angle A'A_0N$.

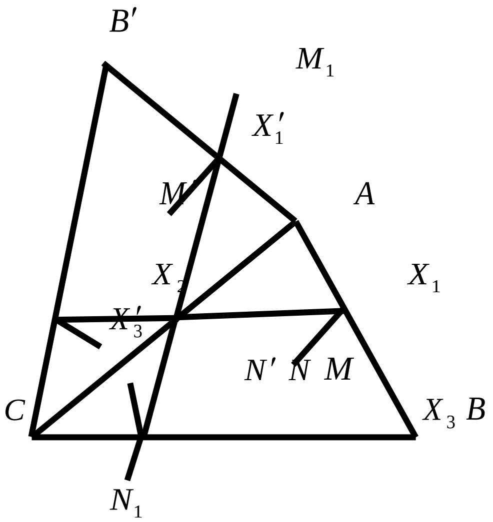
$\angle AA_0M = \angle A'A_0N$ $\angle A_1A_0N = \angle A'A_0N$ From the equation, we find that the. $\angle AA_0M = \angle A_1A_0N$

allied. $\angle BB_0M = \angle B_1B_0N, \angle CC_0M = \angle C_1C_0N, \angle DD_0M = \angle D_1D_0N$

target 9.

ABC M,N $MX_1X_2X_3N$ *Points are given within the triangle. Conduct minimum length of Lamaism, there.* $X_1 \in AB, X_2 \in AC, X_3 \in BC$

amusement:



$\Delta AB'C$ ΔABC AC M We will build, symmetrical, relative cuts. Then the point will correspond to the point. We will construct a point, the symmetry point, and a point, the symmetry point. M' M_1 M' AB' N_1 N BC

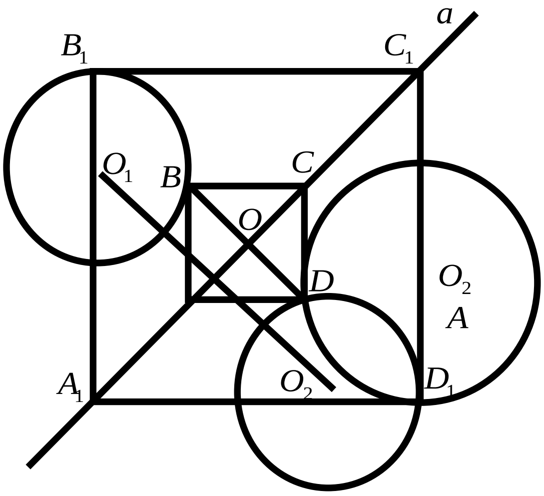
What is clear is that the length is cut depending only on the other M_1N_1

A, B, C, M, N $MX_1X_2X_3N$ $MX_1 + X_1X_2 + X_2X_3 + X_3N$ $X_1X_2 = X_1'X_2$ X_1' X_1 AC
$X_1M = X_1'M' = \; = X_1'M_1, X_3N = X_3N_1$ $MX_1X_2X_3N$ $M_1X_1' + X_1'X_2 + X_2X_3 + X_3N_1 \geq M_1N_1$ Point placement. The horn port length is equal. Considering (when the points correspond to their relative symmetry), we find that the layer length is equal. Obviously, the trumpet is is length when the point is in a straight line. M_1N_1 X_1', X_2', X_3' M_1N_1

Task 10.

Construct a square where two opposite vertices belong to a given line and the other two are given two circles.

amusement:



(O_1, r_1) (O_2, r_2) Let the square of the ABCD —— search, vertices A and C directly belong to the given A, and vertices B and D belong to the given circular i, respectively. Because the diagonal of the AC square is its axis of symmetry, the points B and D are symmetric relative to A, and we construct A circle. Because and $(O_1^{'}, r_1) = S_a((O_1, r_1))$ $D = S_a(B)$ (O_1, r_1) $(O_1^{'}, r_1)$ Top B belongs to Coke, and top D belongs to Coke. In addition, the vertex of D belongs to a circle. So the dot D is the intersection of the circle and. When we construct point D, we also find the intersection of point O-tangent BD and line A. Then OB = OD = OA = OC, and both points A and C belong to A. (O_2, r_2) $(O_1^{'}, r_1)$ (O_2, r_2) $B = S_a(D)$

(2) Same-state method

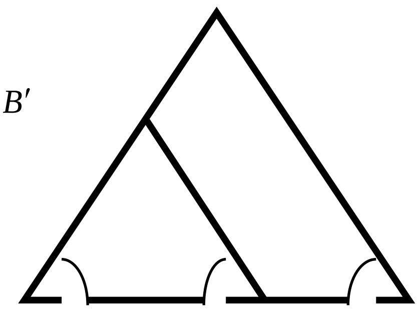
Homomorphism is often used to decompose geometric problems into proofs and constructions. If the nature of the homomorphism is used in deconstructing this geometric problem, then the problem is deconstructed by homomorphism methods. Using this method, the problem of conditionally including the shape and dimensions of the search graph is resolved. Regardless of the size data, we have a task to study some graphs homology with the search object. After studying whether to construct this graph, we correctly selected the homomorphic center and determined its coefficients based on the dimensional data. Then, the searched figure will be the image of the figure already found in the selected homomorphism.

Consider a few examples:

Task 1.

Construct a triangle both along the perimeter and at two angles.

amusement:



construct an arbitrary triangle of Avs at two given angles, similar to the search of.

$AB'C'$ $2p'$ Lets mark the circumference of the triangle. If now take point A as the homomorphic center

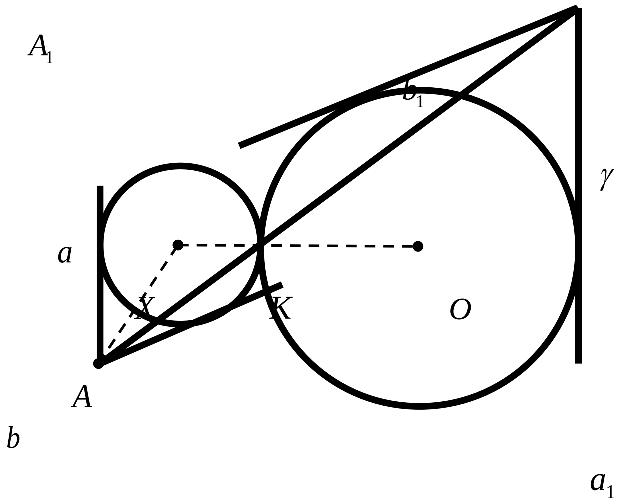
Let the AVS triangle search and its perimeter be set at 2R. We

$k = \frac{p}{p'}$ ΔABC H_A^k With a coefficient, we treated the searched image as a triangle of Avs PRI. Quests always have the only entertainment.

Task 2.

Construct a circle in contact with a given circle and two non-parallel straight Y phases. a b

amusement:



Let- -A centrally centered set circle. Search circle homology circles relative to their contacts, homology is shown on the line, parallel to them and tangent to the circle. The intersection of the let- -straight line.. γ K a,b a_1,b_1 γ A_1 a_1,b_1

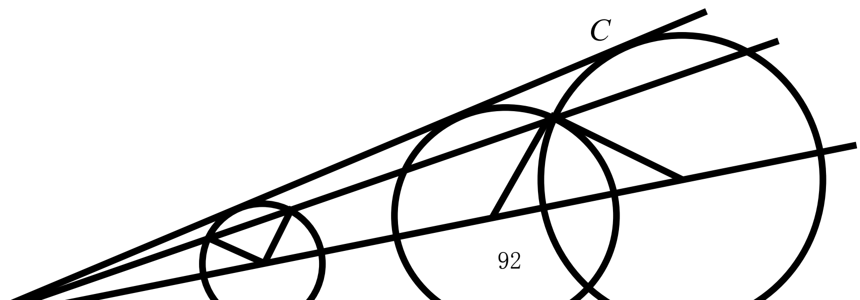
A a,b γ O Marked by the intersection of a straight line.

AA_1 So the straight line passes through the homomorphic center-point. The center of the searched circle is located on the angular and straight bipartition. K X A KO

Task 3.

$\angle ABC$ *Now, enter a circle that passes through a point, which lies within a given angle.* M

amusement:



M

Q P O_2

O O_1

B A

O_2 O_2 $\angle ABC$ O Let a center-search. The center of the circle is located in the bipartition. Lets write any center into the corner. The circle is like a search so it has to be moved so it touches both sides, through the point. Connect the points with them, directly to the intersection of the circle, forming a pair of corresponding points intersect with them. The center of the two homogenous circles will also be the corresponding point. The radius of a circle is the homomorphic radius of the circle, and hence. Thus, the center of a searched circle can be found as the intersection of the double partition with the line.. $\angle ABC$ M B M BM BM O O_2 M Q MO_2 O_2 QO O $MO_2 \| QO$ O_2 $\angle ABC$ $MO_2 \| QO$

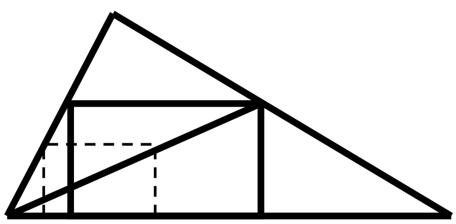
target 4.

The AVS triangles are shown. Enter a given triangle of a square so that its two vertices lie on the basis of the triangular AS, and the other two lie on both sides of the AB and VS of the triangle.

amusement:

We construct a square of the CMNR so that its two vertices K and R lie on one side of the AS and the third M on one side of the AV. Lets take the rays. Let. $N' = AN \cap BC$

B



$M' \; N'$
$M \; N$

$H_A^k \; k = \frac{AN'}{AN} \; H_A^k(KM) = K'M', \; H_A^k(MN) = M'N',$
$H_A^k(PN) = P'N', \; H_A^k(KP) = K'P' \Rightarrow$

$A \; K \; K' \; P \; P' \; C$ $\quad H_A^k(KMNP) = K'M'N'P'.$

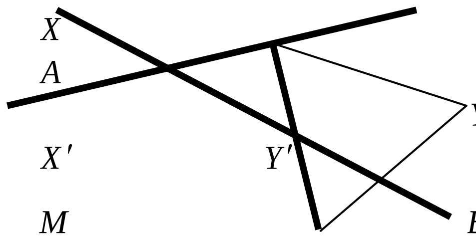
Consider that, yes, and then

The problem has a unique solution unless any of the angles underlying the AS is blunt. If one of the horns is stupid, then the question is no fun.

target 5.

$AA', BB' \; M \; M$ *Two lines are given, which intersect and a point that is not on either one, and the lines are passed through the points so that they directly intersect the data, and the distance between the intersections is equal to the distance from one of these points.* M

amusement:



intersect, the points do not belong to any of them, assume- -search the line, take the points as the center, and- -a pair corresponding $MX \; M \; X \; X'$

$AA', BB' \quad M$ Let the lines

Homostatic point. It is easy to determine as an intersection with a line, which is an image of a line at homology. $X \quad AA' \quad XY \quad BB'$
$k = MX : MX' = 2$

From here you can see how to call the task. Exexport by a straight line, the intersection with which is indicate through. Next, we construct the image of the point (delayed cut) —— point.

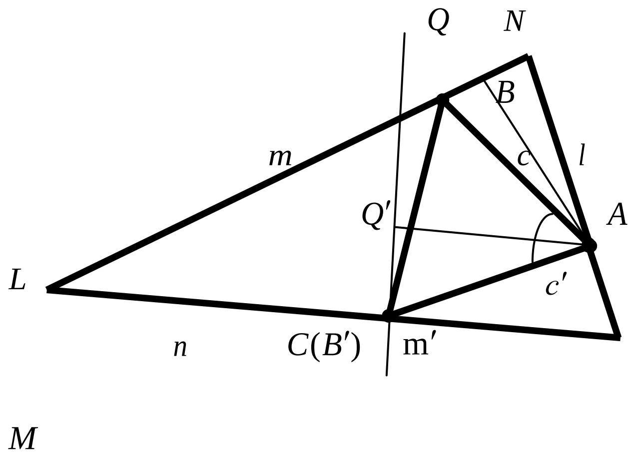
By the conductor passing at the point. Direct search. M MY BB' Y' Y $Y'Y = MY'$ Y' Y $YX \| BB'$ AA' X MX

(3) Rotation method

Task 1.

l,m,n l A *Three straight lines and the points are given. Construct an equilateral triangle such that its vertex lies on three straight line data, one of which-at the point. A*

amusement:



ABC m',c' m c Let the triangle search. Find the image of the lines and edges of the triangle determined by the center and angle (counterclockwise). The dot image will be a dot matching the dot. A $\alpha = 60^\circ$ B B' C

A AQ m AQ' 60° AQ AQ' AQ Q' From here, you can see how to entertain the task. From the point, we look perpendicular to the line and build lines on it. Next, we delay a cut =, and, at a point, a line that intersects a straight line perpendicular to the point. Finally, we find images of the turning cuts determined by the center and angle (clockwise motion). Lets find the vertices of the sought triangle. n C AB AC A $\alpha = -60^\circ$ B

Task 2.

A triangle is constructed on both sides of the I and the median. a b m_c

amusement:

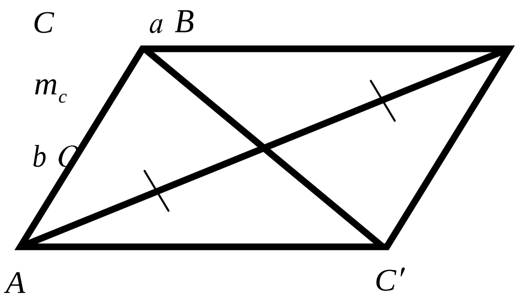


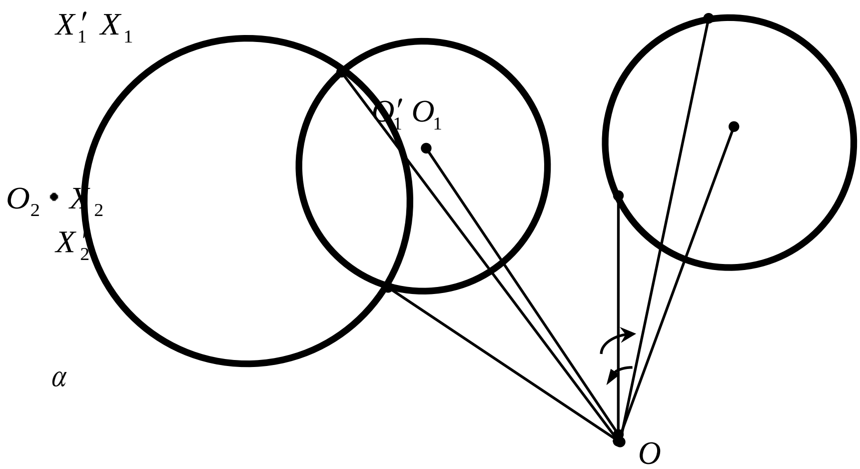
image is determined by the center and rotation angle (counterclockwise), we get a parallelogram, where $B'A'C'$ O $\alpha = 180^\circ$ $AC'BC$

ΔABC OC Let-search and-his median. If you consider that its CC' $\Delta ACC'$ All edges and diagonal lines are known. So all three aspects are well-known. If we build it, we will find a search triangle.

Task 3.

O O *Two circles and dots are given. A circle is described by taking a point at the center of the circle, such that the cross section defined by its intersection with a given circle can be seen at a given angle.* O

amusement:



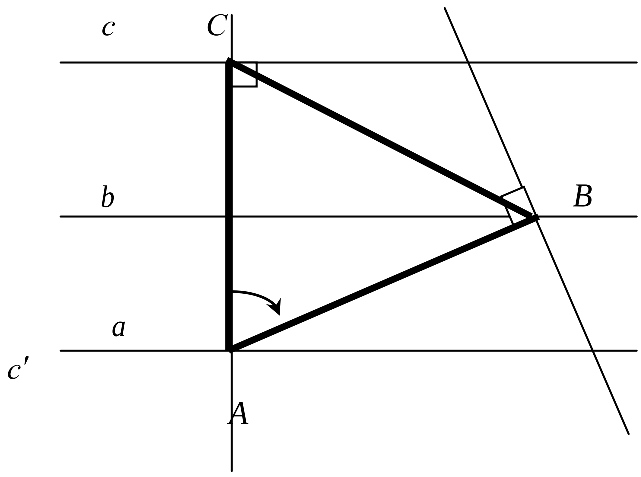
$(O_1,r_1),(O_2,r_2)$ O O O α (O_1,r_1) (O_2,r_2) O α Let-round the data and-given the point. The points searched are located on a circle with the center at that point, and the cuts defined by them can be seen from a given angle. Therefore, they belong to different circles & correspond at the turn, determined by the center and turn angles. From here: we find an image of the rotating circle, determined by the center and angle of the rotation. The circle of the center is the shape of the center. We find the point, and correspond to it. O α O_1' O_1 X_1' X_2'

The task has one partition, two partition, or no partition at all.

Task 4.

a,b,c ABC Three parallel straight lines are shown. Construct an equilateral triangle where the vertex is located directly on the data.

amusement:



a,b,c Let- -three parallel straight lines. Select any point to direct. When you turn straight into a corner, her image goes straight up. As a point lying on a straight C, then when a straight C point turns, the C will belong to a straight. And this point will be at the intersection of the line A $C\ c\ c-60^{\circ}\ c'\ C\ c'\ c'$

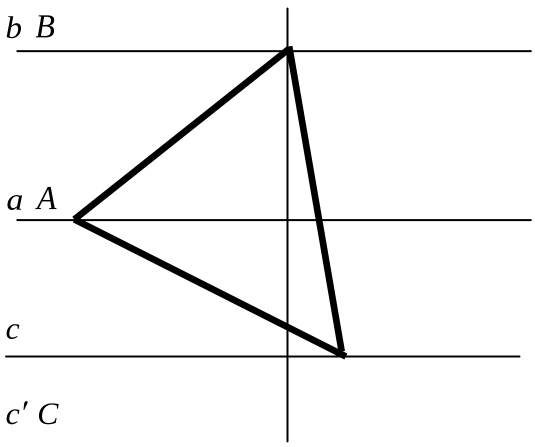
$C\ B$ Will move to the sum of points formed by the intersection of straight lines. So we get the two sides of the triangle. In a circular circle, equal to the circle of the side, we make a circle on the side. As a result, we form an equilateral triangle. $b\ c'\ AC, AB\ B$

$AB\ AC$

target 5.

Construct an isosceles triangle whose vertices belong to three given parallel lines.

amusement:



Lets do it: AAH BBYC.

Suppose that the ABC triangle is constructed to satisfy the task condition. We chose a turn (counter-clockwise direction). then (C). We will build directly from (c). Point C belongs to direct C and thus $R_A^{90^0}$ $B=R_A^{90^0}$

$c_1=R_A^{90^0}$

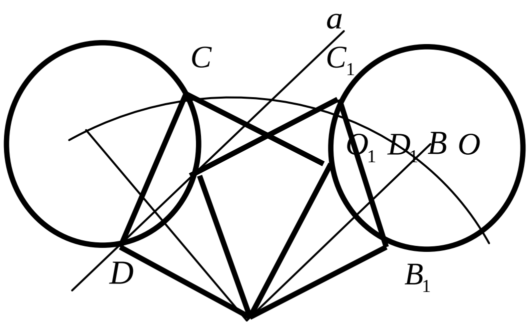
Point B belongs to straight c_1, But point B is also a direct B; so point B is the intersection of direct B and c_1. With B, well find B.

$C=R_A^{-90^0}$

Task 6.

A square is constructed such that its vertices match the given point, and two adjacent vertices-one is located on a given circle and on a given line.

amusement:



Given: A-point, A-straight, (0, r) -round. Structure: ABCD is a square, where point B is circular, (0, r), and point D is A straight line A.

Let the ABCD be the square that satisfies the task condition. optional

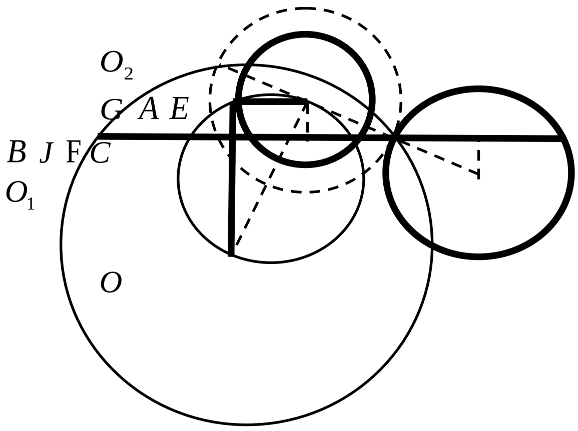
$R_A^{90^0}$ Turn (counterclockwise) and then. We will build a circle. Point B belongs to the circle (0, r), because point D belongs to the circle (0 $D = R_A^{90^0}(B)$ $(O_1, r) = R_A^{90^0}((O, r)$ *1*, R), but point D belongs to A, then D is the intersection of the line A and the circle (0*1*, *r*) 。

$B = R_A^{-90^0}(D)$ If a have D, we will find that B belongs to Coke (0, r).

Task 7.

A O, O_1 BAC $|BA| - |AC|$ *By the intersection of the data, the difference of the length is equal to the given intersection.* $2a$

amusement:



(O_2, O_1A) O_1 A $\alpha = 180^\circ$ Let the circle be the image of the circle at the center, determined by the center and the rotation angle (counterclockwise). That way, Hody would be Hoda. We are building it. Then, then. AC AJ $OD \perp AB, O_2F \perp AJ$ $O_2J \| DF$ $O_2G = DF = a$

O_2GO OD O_2G This way we can construct triangles and define straight or straight lines. Lets come to this building. In the direction of the continuing incision we will delay a point and on the incision we will establish a circle. We describe a length up to the length of the intersection, with a wheel. The January of the search will be. O_1A A O_2 $AO_2 = AO_1$ OO_2 (O_2, a) G, H $AB \perp OG, AB_1 \perp OH$

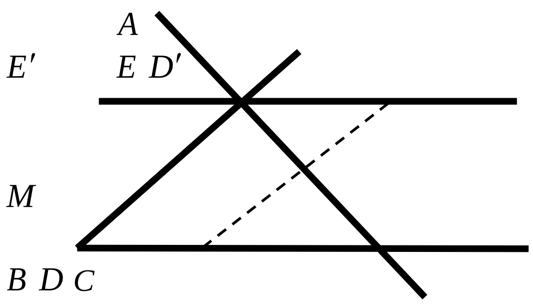
To prove the correctness of the construction performed here, note that the string A. Tom. But, and. $AJ = AC,$ $AF = AE$ $|AB| - |AC| = 2(|AD| - |AF|) = 2|DF|$ $DF = O_2G = a$ $|AB| - |AC| = 2a$

(4) Centric symmetry method

Task 1.

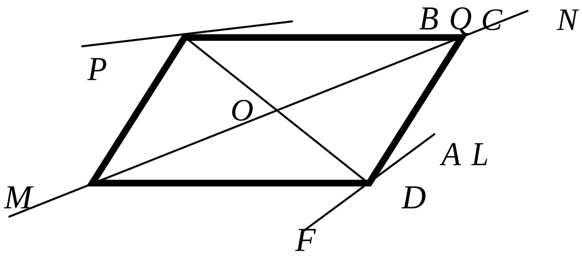
ABC M An angle is given, and its inner point is arbitrarily given. Through this point, proceed directly so that the cut on both sides of a given angle is split in half by the point.

amusement:



$E'D'$ BC M . Construct a symmetrical edge of a line, relative to a line of that point, by any line that intersects the point at that point. M MD BC D MD We then delayed the incision $MD' = MD$ $E'D' \| BC, (E \in AB)$ E $E'D'$ AB And well do it directly. The intersections and edges belong to the line being searched.

Task 2.



on a given line and its two vertices on two lines, different from the data. PQ, FL MN

amusement:

AC MN *Construct a parallelogram with a diagonal line*

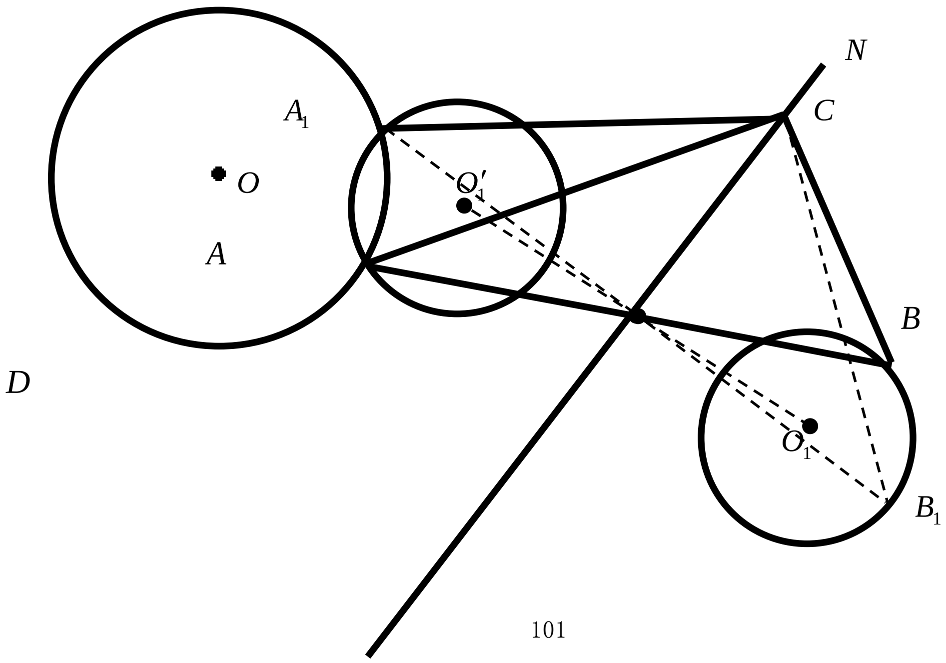
Through the middle cut allows us to make a straight line crossing O AC BD PQ, FL

B, D BD O Points, so that the cut is divided to half (see the previous task). It is easy to determine what is the parallelogram of the search. $ABCD$

Task 3.

$(O,r),(O_1,r_1)$ MN *A straight line was performed between the circles, where the incision was delayed. A triangle is constructed such that the tangent is the median and the vertices are located on the circle.* CD ABC CD A,B $(O,r),(O_1,r_1)$

amusement:



M

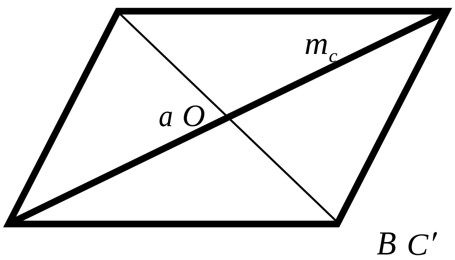
(O_1',r_1) (O_1,r_1) D A,A_1 (O,r) (O_1',r_1) AD,A_1D (O_1,r_1) B,B_1 B A B_1-A_1 D ABC A_1B_1C Construct a circle, relative to a given circle with point symmetry. Then through the intersection of the circle, they intersect the circle at the point. Clearly, the points are symmetric and are relative. Triangles and the search. The task has one, two slides, or no sliding at all. This depends on whether the circle and intersect, contacts or disintersect. (O,r) (O_1',r_1)

Task 4.

A triangle is constructed on the two sides and behind the median. a,b m_c

amusement:

ABC Hypothesis-Search for triangles and their medians. $m_c=|OC|$



$ACBC'$ A parallelogram in which all edges and diagonal lines are known. So all three new aspects are known. formation CC' $\Delta ACC'$

$\Delta A'B'C'$ ΔABC Lets construct the relative symmetry points. We will be getting there. O

B A CC' O This triangle, we find a point, relative to the tangent (point) center symmetry. So we found all three vertices of the search triangle.

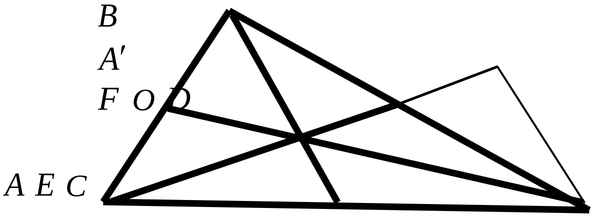
If the slice is conditions, the task is feasible:. a,b,m_c

$a+b>2m_c$, $|a-b|<2m_c$

objective 5.

Construct a triangle with its three median values. m_a, m_b, m_c

amusement:



reminds that the median value of the triangle intersects at a point, and O

ABC $m_a=|AD|, m_b=|BE|, m_c=|CF|$.

The let-out median triangle

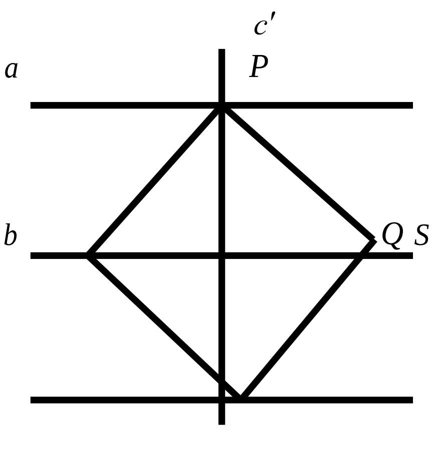
A' A O $\Delta A'OC$ $|OA'|=\frac{2}{3}m_a$, $|OC|=\frac{2}{3}m_c$, $|CA'|=\frac{2}{3}m_b$. If you construct a point, relative to a symmetric vertex of a point, then we will get a known edge: we construct this triangle, and at its edge, we will find a relatively symmetric point. Next, we delayed the incision to one side and proceed directly. We delayed the incision up to this straight line. Triangles is the search. OA' A A' O CO $CF:|CF|=m_c$ AF F $FB=AF$ ABC

If the cut data can be taken from both sides of the triangle, then the problem has a single decoupling. m_a, m_b, m_c

Task 6.

Construct a square so that its three vertices lie on the three data in a parallel line. a,b,c

amusement:



c

R

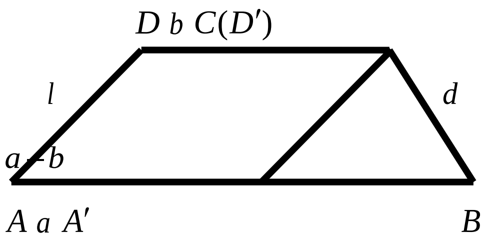
$P\in a, Q\in b, R\in C$ $a\cap c'=P$ Let PQRS be a square search, one. When turning around point Q at 90°, on. By constructing the cut PQ point R coincides with point R, (the side of the square), we will and when line C turns to line S, easily construct the entire it passes through point R, and so square.

(5) Parallel transfer method

Task 1.

Build a trapezoid on her four sides. a,b,d,l

amusement:



looking for the trapezoid, with. Consider the parallel transfer with the border shape. $\overline{DC}$ AD

$ABCD$

$|AB|=a,|DC|=b,|BC|=d,|AD|=l$ Jean-

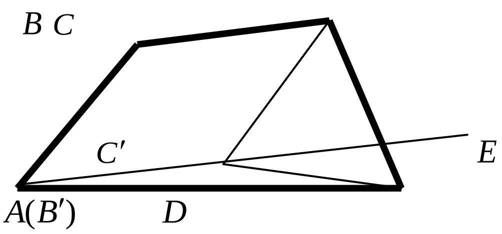
$\Delta A'BC$ $|A'C|=l,|BC|=d,|A'B|=a-b$ We will have some familiar parties. To construct the searched trapezoid, first construct a triangle on its three edges and then find the image of the edge in parallel. $A'BC$ $A'C$ $\overline{CD}$

If a triangle could be built over the three incisions, the task is possible. $a-b,d,l$

Task 2.

Construct a quadragon with its four corners and a pair of opposite edges.

amusement:



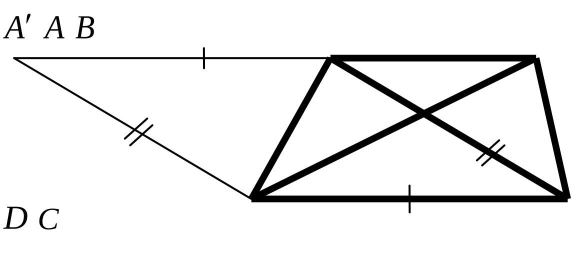
sides in the quadrilateral. Consider parallel transfers and the resulting image of the party. Reeach a triangle in which the sides are known and, $\overline{BA}$ $B'C'$ BC $\Delta ADC'$ AC', AD $\angle C'AD$

$ABCD$ A, B, C, D BC, AD Suppose the known angles and opposite $\angle BAD - \angle BAE$ The value is equal to the difference between the values of the two known angles. Such triangles are very easy to establish. A line can be followed because the former forms a known angle with the line, while the latter is a known angle with the edge. Still to be performed and. $C'C, DC$ $AC'(\angle CC'E = \angle FBC)$ CDA AD $CB \| C'A$ $AB \| C'C$

Task 3.

Construct a trapezoid, with its diagonal lines and two parallel edges.

amusement:



$ABCD$ AC, BD AB, DC $AB \| DC$ Let both the oblique Angle and the edge in the trapezoid know, yes. If a parallel transfer is considered and found $\overline{CD}$

CA $\Delta A'BD$ $|A'D| = |AC|, |A'B| = |A'A| + |AB| = = |DC| + |AB|$ BD When it cuts the image, you can get a known edge, a known diagonal line. By constructing this triangle, we will find the image of its edges as they move in parallel. Lets mention it. The trapezoid was searched. $A'D$ $\overline{A'A}$ $|A'A| = |DC|, A \in A'B$

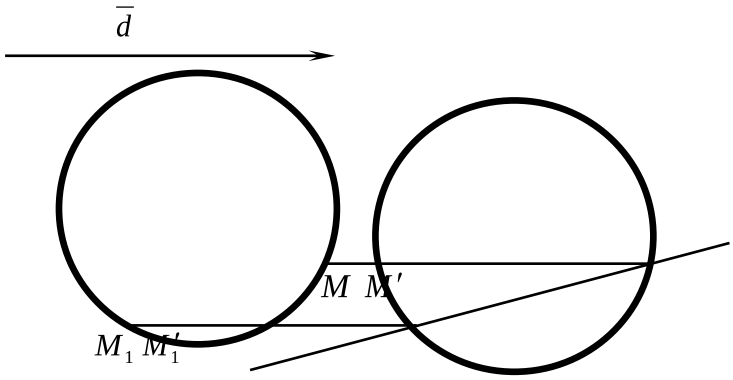
If you perform a ratio to the tangent, the task is possible:. AC, BD, AB, DC $\left||AC| - |BD|\right| < |AB| + |DC| < |AC| + |BD|$

Task 4.

Between the line and the circle, make a directional cut that represents a given vector. $\overline{d}$

amusement:

$\bullet\, O \bullet O_1$



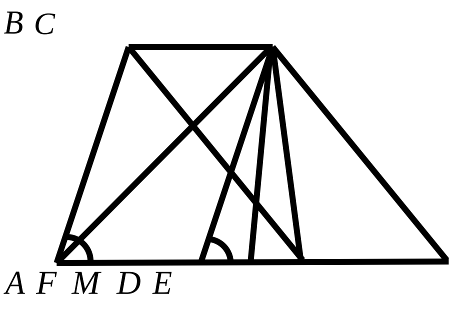
Find the image of a circle in parallel: the center of the circle is the image of the center. $\overline{d}$ O_1 O

The task may have one, two, or none at all.

target 5.

Construct a trapezoid, two diagonal lines on the base.

amusement:



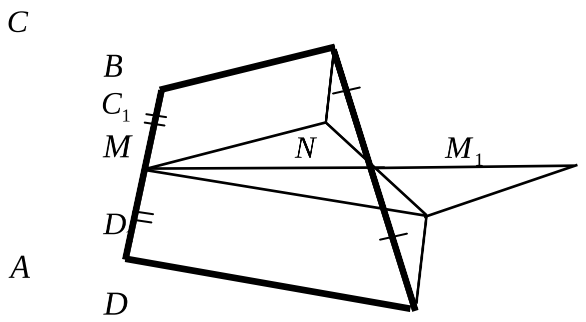
$ABCD$ Let the trapezoid be built. Perform parallel transfer at the side points and diagonal transfer at the points. Lets consider the triangle. C AB BD C $CFD(CF \| AB, \angle CFD = \angle BAD)$

$CM \ ACE(AF = BC = DE)$. The median of this triangle will be the median of the triangle that can be precisely constructed similarly and in which the triangle will find the angle between the median and the edge. Now, to build a triangle, at the angle between side and side. $CFD \ \varphi \ ACE \ AC \ AE \ \varphi \ CM \ AE$

Task 6.

Constructs a quadrangle ABCD in an MN cross section between its edge and one connecting the AB and CD edges.

amusement:



Let search the ABCD quadragonal construct, where M is the middle of the AB section and N is the middle of the CD section.

To bring the BC and AD incisions closer, parallel shifts were performed

The BC and AM vectors were used separately for BC and AD cutting, namely construct and. Then (Bo BM = AM = DD $MC_1 = T_{\overline{BM}}(BC)$ $MD_1 = T_{\overline{AM}}(AD)$

$\Delta CC_1N = \Delta DD_1N$ *1* $\angle C_1CN = \angle D_1DN) \Rightarrow$ $\Rightarrow (\angle CNC_1 = \angle DND_1$, CN = ND,), so the point C.*1*, *N*, *D1*Belongs to a direct and a $C_1N=ND_1$.

Parallel transfer of the MC incision was performed*1*vector MD*1* $D_1M_1 = T_{\overline{MD_1}}(MC_1)$:. Yes:, from MN = NM $\Delta MC_1N = \Delta D_1NM_1$ *1* ΔMM_1D_1 In all aspects of him. Thus, the problem boils down to constructing a triangular MM*1*D*1*On the famous three sides. With this triangle, it is not difficult to construct vertices searching the quadrilateral.

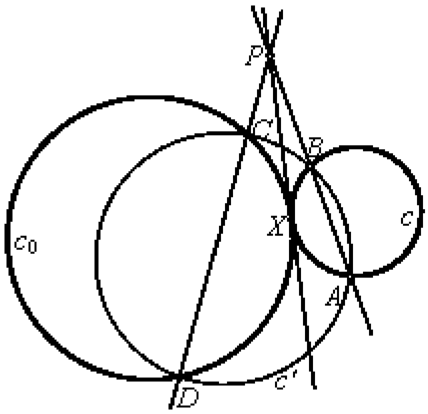
(6) Inversion method

Inversion methods offer the possibility to solve a series of the most difficult design problems in elementary geometry. In doing so, it is combined with the coordinate method, which in practice when trying to solve the problem in the complex plane, gives the most accurate calculation of the location of the desired graph, with obvious advantages over the rather imprecise manual construction. The disadvantage of this approach is its tedious due to the need of rather tedious computations.

Task 1.

Construct a circle with data passing through points A and B and touching on a given circle. c_0

amusement:



intersection of lines AB and CD at points C and D. Next we will mark by X point by P tangent to the given circle. Circle, c through A, B and X will then be searched.

c_0 P is marked by arbitrary circles C ′ and B and by the

Indeed, the degree of the point P is equal to the PXc_0 ^{2}=PC•PD。 Applying the degree theorem to the circle c ′ and its two homomorphic PCD and PAB, we find that PCPD = PAPB. From this deduction, the PX·· ^{2}·= PAPB, so the contact of direct PX and circle c is the common contact circlec_0

c_0 c_0And c. so c touches (at point X). From point P, two tangents can be made, and they will give us two clues to our task.

Mission 2. (Apolonia Mission)

Construct a circle, which touches on the three data of the circle. K_1, K_2, K_3

amusement:

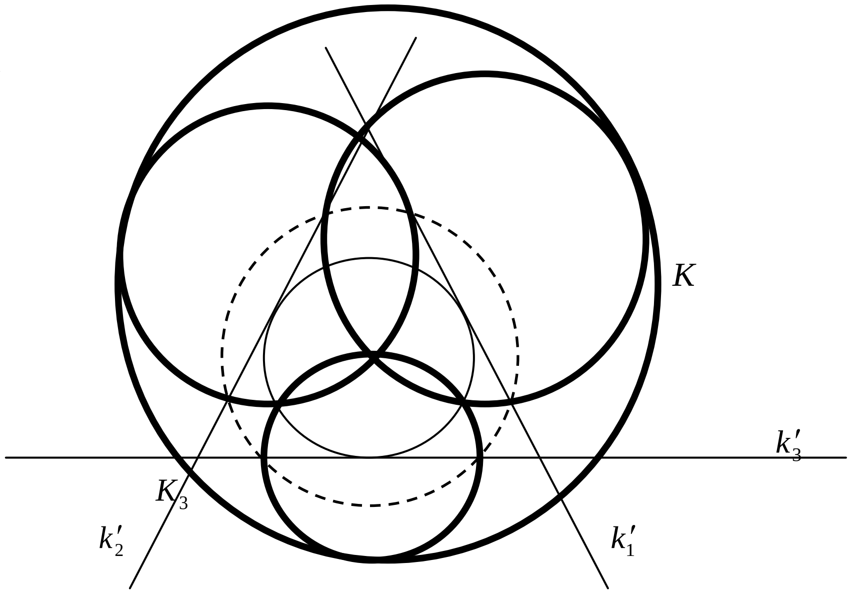
Consider the data of a circle, intersect, through a point, and each of these circles is located outside of the other two, and the radius of one is smaller than the radius of the other two.

K_1, K_2, K_3 O K A) Let the three intersecting circles pass through a point, and be a search circle. Take a circle with any radius as the inversion circle. Then the image of the circle will be straight, intersecting at the point- -different from the intersection of the circle. O K_1, K_2, K_3 k_1' k_2' k_3' A, B, C O

K_2 A K_1

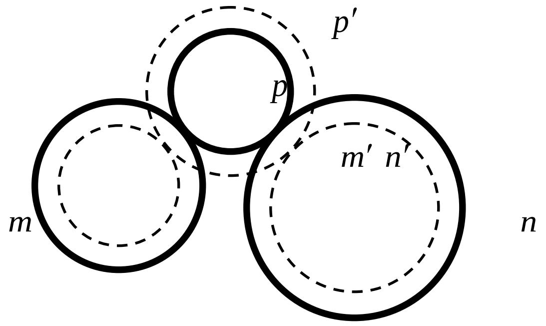
O

K'
B C



K ΔABC In this way, a circle will be a circle written in it. We have come to such buildings from here. A circle with a radius of a center at a point. Assuming that the circle is determined by the inversion, we construct three straight,,, -circles of the data images. We write the circular B that is straight and find its image. The circuit will be searched. O k_1' k_2' k_3' K_1, K_2, K_3 K' ΔABC k_3' k_2' k_3' K K

K'Note that the circle itself is not worth building. It can be confined to the points that construct it to the edges of the triangle, find the images of these points, and make a circle through them.

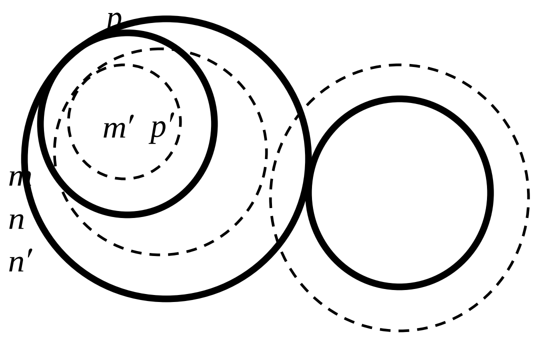


And b) before we consider another track for the Apollo mission. Let a circle, and touch a circle. If so.alike K_1, K_2, K_3 m,n p p m,n

Contact (outside or internal), then as the radius of all mouth increases or decreases, the radius of the circle will also decrease or increase to the same incision. m,n

p m n When the same Z has an intrinsic touch and Z has an extrinsic touch (or opposite), from

By increasing or decreasing its radius to a certain section radius of a circle

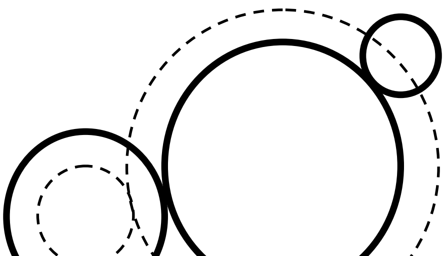


m,n One increases, the other decreases.

Now let us proceed with the mission itself. Suppose that a circle with a radius for our placement K_1, K_2, K_3

$r_1, r_2, r_3 (r_1 > r_3, r_2 > r_3)$ And center respectively. The task is completed and is a search circle. O_1, O_2, O_3 K

O_3



K K_3

O_1
K_1''
K_1
K_2'' O_2 K_2

K_3 K_1 K_2 If the circle radius decreases, then the radius of the circle increases or decreases, depending on the touch nature of the circle. On our drawings, the radius of the circle, it will decrease. Of course, when the same cut, the round K_1 K_2

K Touching on all of these altered circles. Clearly, the radius of the circle is also changed to the same incision. K

K K_1 K_1', K_2'' $r_1 + r_3, r_1 - r_3$ K_2 K_2', K_2'' $r_2 + r_3, r_2 - r_3$ O_3 K_1' K_2' K_1'', K_2'' So, you can build a circle in this order. Describe two concentric circles with a radius and two concentric circles with a circle. Next, we construct a circle that passes through and has the same tactile properties (external or internal). We will come to the four circles, based on which we will build the four circles of the search (they will be concentric with the established circles). Next, we build four more circles that each pass through and have different properties of touch (one with the outside, the other with the inside). We move from these circles to the wanted people. We will receive entertainment for four additional missions. So that we have eight entertainment venues in total. O_3 K_1', K_2'' K_1'', K_2'

In some cases, Apollos mission was completely unentertaining, especially

K_1, K_2, K_3 K_1, K_2, K_3 If they are in concentric circles. If they go through a point and are touched by a pair, then the task will have a lot of fun.

2.3 Study review

In the history of mathematics curriculum reform, geometry is the focus of the previous mathematics curriculum reform. The course content of compulsory education has graphics and geometry modules, as well as the corresponding course requirements of graphics and transformation. The introduction of the viewpoint and content of geometric transformation into the mathematics curriculum of basic education in China not only shows the importance of the content of geometric transformation in China, but also accords with the development trend of mathematical science and international mathematics education.

Graphics and transformation are important teaching carriers to develop students spatial concept and geometric intuitive ability. Geometric transformation enables students to experience the transformation process from concrete and intuitive to abstract graphics, which is the preliminary cultivation of college students spatial concept. The introduction of geometric transformation changes the geometry from static state to motor change state, which to a certain extent imperceptibly cultivates students geometric intuitive thinking and deepens students understanding of geometric knowledge.

2.3.1 Research on the value of geometric transformation ideas

In 1982, "Primary and Secondary School Mathematics Teaching Syllabus" stipulated the teaching content as follows: teaching should follow certain rules, conform to the law of students cognitive development and acceptance ability, from easy to difficult, from shallow to deep, and gradually infiltrate some new mathematical ideas and methods. For example, the collection and corresponding ideas are properly permeated into the textbook.

Yao Zhenqian pointed out that geometric transformation is an important thought in modern mathematics. In the initial stage of geometry learning and the in-depth stage of geometry learning, the idea of geometric transformation should be constantly permeated[6]. Textbooks also attach great importance to the infiltration of the idea of geometric transformation. The folding and jigsaw puzzle in the course of plane geometry are in essence a transformation method. Therefore, teachers should first understand the writing intention of the infiltration of the thought of geometric transformation, and pay enough attention to it in teaching. For example, before talking about the judgment of the equal triangle, we can first discuss various forms of contract transformation, and the patterns in the textbook can be summarized and classified according to translation, rotation and axisymmetry, which can not only stimulate students interest in learning, but also help students understand the role of geometric transformation.

Gu Tingyu and Yang Chongming pointed out that the use of geometric transformation can be traced back to the root of the common methods of geometric problem solving. Teachers explain the essence of test preparation to students through geometric transformation, which can not only deepen students understanding of the problem, but also improve students thinking quality[7]. Wang Jinggeng puts forward the idea of geometric transformation in university geometry teaching, which provides a new and more effective method for geometric argument and creates certain conditions for the establishment of the connection between university mathematics and modern mathematics[8]. Ma Zhimin and Chen Tianzhu pointed out that the idea of geometric transformation is an important idea in modern mathematics, which is convenient to grasp the transformation of variables, simplify the idea of solving problems, so as to quickly find the solution to the problem[9]. Cheng Chuanping believes that geometric transformation makes the research perspective of geometry shift from static to motion, and transformation deepens the students understanding of graphics, and transformation is also a method of argument[10]. Therefore, in teaching, teachers should start from the reality, gradually permeate the idea of geometric transformation, and gradually apply the transformation method to the process of geometric proof.

Shi Ningqiang in the Plar Geometry Transformation Plan[11]The following discussion: university plane geometry trains students geometric argumentation ability, but also trains students geometric intuitive ability. The cultivation of mathematical intuitive ability comes from students deep understanding of the nature of mathematical objects and students accumulated experience in mathematical activities. Geometrical intuition is the premise of geometric argument, and geometric transformation helps students to understand the essence of geometry and form geometric intuition, therefore, geometric transformation is also the basis of geometric argument. The visualization, visualization and dynamics of university plane geometry teaching is also the main goal of plane geometry reform. The basic idea of the reform is to define geometry on the basis of translation, rotation and reflection transformation in the university stage, and to use two-dimensional matrix to express geometric transformation in the high school stage. It is convenient to discuss orthogonal transformation in advanced mathematics and group theory in approximate algebra. Due to the wide application of changing ideas, students can learn changing ideas even if they do not choose mathematics major after finishing the university, which is also very helpful to the study of physics, chemistry, biology, computer and other subjects. Professor Shi Ningzhongs view provides ideas for the infiltration of the infiltration of geometric transformation in university teaching, which also lays a certain theoretical foundation for this research. And how to establish the university plane geometry teaching on the basis of geometric transformation theory, is a need to be deepQuestions into the study. Through the study of geometric problems solved by using geometric transformation method in the new curriculum standard, Tang Yongdong emphasized the importance of geometric transformation in cultivating students spatial concept[12]。

Qiu Weiping discusses the main functions of three geometric transformations: providing corresponding basis and method for determining the equality of figures, guiding ideology and dynamic analysis method for the theory of plane geometric exercises[13]. All these fully explain the value of geometric transformation, and should be paid attention to and relevant research in teaching. Geometric transformation can be used as the basis of argument, and its application is beneficial to broaden students ideas of solving problems[14]。

About the value of geometric transformation in the high school entrance examination, Hao Zhigang pointed out that the high school entrance examination takes geometric transformation as the background, to examine the students comprehensive geometric knowledge, and the focus is to examine the students comprehensive ability to independently explore, analyze and solve problems[15]This requires teachers to pay attention to the teaching of geometric transformation in teaching.

Through the study of African folk mathematics, Tang Hengjun found that geometric patterns including translation, rotation, reflection (axisymmetry) and similar transformation are prevalent in the daily necessities of African people[16]. This phenomenon is also reflected in

Chinese traditional culture, such as Chinese paper cutting, carving, architecture and other arts, so it can be seen that geometric transformation has important artistic value.

Taking axisymmetric transformation as an example, Shen Xuemei discusses the important value and infiltration strategy of the idea of university geometric transformation in expanding students thinking level[17]In teaching, we should make full use of teaching molds to deepen students understanding, pay attention to the effective setting of problems, and improve students meta-learning ability, which not only reflects the value of geometric transformation thought in university geometry teaching, but also provides a certain reference for the infiltration of geometric transformation thought in university geometry teaching.

In Dai Changlongs opinion, in the face of ever-changing geometric problems, the idea of geometric transformation can help students master the solution of a kind of geometric problems, can guide students to think from multiple angles, so as to help students explore the ideas of solving problems. For geometry teaching, the key to the transformation idea is that there are some invariants before and after the transformation, and the method of geometric transformation to establish a new quantitative relationship[18]This kind of thinking can cultivate students divergent thinking.

Xu Jiaxiong proposed to introduce geometric transformation into European geometry, that is, regard geometric figures as both an object and a process. Understanding Euclidean geometry through the perspective of dynamic transformation can help students understand the connection between knowledge and methods, so as to grasp the deep structure of the problem and improve students ability to solve problems[19]。

Ren Zhiyan believes that the penetration of geometric transformation in the university stage not only helps students to find the internal essential connection between graphics, but also contributes to the development of students thinking[20]. In the college stage, students thinking ability of geometric transformation has been improved, and students understanding of geometric transformation knowledge has been deepened. On this basis, students can have a deeper level of independent thinking. Only by forming the habit of independent thinking can students lay a good foundation for their future mathematics learning. How to penetrate into the teaching practice? All of these will need to be studied in depth.

Luo pointed out that motion is called transformation in mathematics, and European geometry mainly studies isomorphic and similar forms[21]. Among them, the contract transformation keeps the length of the line segment unchanged, and the similarity transformation keeps the size of the angle unchanged. These two transformations are the mathematical background of the graph transformation in the university.

In "Geometry Transformation and Geometry Certificate"[22]In detail, Xiao Zhengang introduced the concept, properties and its application in the geometric transformation, which is an effective tool to solve the problem of plane geometry. By transforming geometric figures, the original scattered conditions can become more concentrated, so as to achieve the effect of simplifying complexity, turning difficult into easy, and improving efficiency.

Liu Yali believes that, as a kind of modern mathematical thought, the idea of geometric transformation is to use the point of view of movement and transformation to study geometric figures. In the process of movement and transformation, there will be many invariable properties and invariants, and geometric transformation is an important means to establish this relationship[23]。

In short, geometric transformation is of great value to the understanding of geometric figures. Movement and change is a universal state of all things. In this state, the nature and relationship of geometric figures can be grasped. Geometry transformation is an effective and powerful tool to solve geometric problems. The learning of geometric transformation thought can cultivate students thinking mode of movement and change. Geometrical transformation provides a certain background for geometric exploration, promotes students deep independent thinking, and lays a foundation for future learning.

Therefore, in the teaching of university geometry, the idea of geometric transformation should be permeated into the plane comprehensive geometry, which is a comprehensive static comprehensive geometry movement, so as to increase the interest and logic of plane geometry, and emphasize the use of geometric transformation to explore the direction of problem solving, so as to provide problem solving ideas for geometric proof. The university geometry course based on the basis of geometric transformation theory is a worthy of long-term research topic, this paper must first study the geometric transformation thought penetration into the actual geometry classroom teaching, and the premise is to understand the current geometric transformation thought, so the present situation of teaching research, according to the actual teaching teaching penetration experiment.

2.3.2 Teaching and research on the idea of geometric transformation

Jin Baoyun proposed that in order to realize the modernization of college geometry teaching, it is necessary to introduce the idea of geometric transformation into college geometry teaching, which is conducive to students forming modern mathematical thought. From three levels[24]Teaching of geometric transformation: (1) to clarify the basic concepts and make full use of the nature of transformation.(2) Establish the mutual relationship between geometric transformation and mapping.(3) The idea of infiltration and transformation group. In view of the improvement of university students thinking development level, it is of great significance to cultivate students to clarify the concept of geometric transformation and permeate the idea of modern mathematical thinking.

Through the compilation and experiment of Shanghai University geometry experiment textbook, Ding Shengbao put forward the outstanding characteristics of the compilation of the textbook: organize the textbook from the perspective of geometric transformation, which can train students to observe and understand geometric figures from the perspective of motion transformation[25], Pay attention to the inquiry under the premise of acceptable to students, and cultivate students logical reasoning ability. Bao Jiansheng proposed three levels in the application of geometric transformation[26], Level 1: the name of applying geometric transformation; level 2: the idea of applying geometric transformation; level 3: the language of applying geometric transformation. In the teaching, we should pay attention to grasp the degree of application, and the requirements of these three different levels are deepening constantly.

Jiang Zongde believes that in the teaching of university geometry, it is not only necessary, but also feasible to consciously contact students with the ideas and methods of geometry transformation. This is not only the need of geometry teaching full of vitality, but also the need of cultivating students dialectical thinking ability and making creative exploration. It is not only helpful to analyze the examples, deal with the teaching materials, and find out the dialectical thinking quality of the teaching materials, but also help to reveal the law of the compilation of the teaching materials[27]。

Bao Jiansheng stressed that the setting of future geometry curriculum in China should highlight the important role of geometric transformation, and grasp the "degree" of geometric transformation from the students actual understanding of geometric transformation[28]. Li Yucheng pointed out that teachers should deal with geometric problems from the perspective of geometric transformation, and emphasized the cultivation of students concept of geometric transformation in teaching[29]。

Zhao Shengchu, Xu Zhengchuan and Lu Xiumin stressed that in order to integrate geometric transformation with university geometry curriculum naturally, it is necessary to choose appropriate knowledge growth points on the basis of students acceptance and understanding, and on the basis of the original geometry curriculum system[30]. For example, the vertical bisector of a line segment is the knowledge growth point of axisymmetric transformation, the parallelogram is the knowledge growth point of translation transformation, and the circle is the knowledge growth point of rotational transformation. In practical teaching, if axisymmetry, translation, rotation and so on as the "new basic geometric mapping" statements are applied to geometric problems, it is easier for students to master the basic knowledge of graphics, so as to better master the basic knowledge of graphics.

Zhao Shengchu and Bai Chengyu believe that the essence of mathematical development lies in using the understanding of geometric courses from the perspective of geometric transformation. Due to their cognitive level, knowledge base and other reasons, college students use superposition to make it one of the effective ways for students to learn, and on this basis, permeate the one-to-one correspondence relationship[31]. Using the idea of geometric transformation to define and judge the equality of triangle can make it easier for students to understand the concept of equality, which promotes the development and construction of geometry discipline to a certain extent.

In Chen Jianxins opinion, there are rich basic mathematical ideas implicit in the geometric transformation and its application process. Understanding the basic idea of mathematics is one of the basic requirements of compulsory education curriculum. How to effectively carry out the equal and similar courses in university mathematics under the guidance of geometric transformation concept in the teaching practice is a problem that needs to be solved especially at present[32]。

Li Chunchang and Yu Xiuyun emphasize the importance of the idea of geometric transformation, explore the mathematical knowledge contained in the graphics, and cultivate students ability of geometric solving problems. In the teaching process of geometric transformation should attach attention to diversified teaching, highlight the basic nature of graphics transformation teaching, the students can stand on the higher view of geometric graphics, from the perspective of graphic transformation to appreciate and design graphics, can understand the beauty of mathematics, using geometric transformation for artistic creation, cultivate the students aesthetic ability and ability to create beauty[33]. Hu Rongping studied the teaching strategy of geometric transformation thought in university mathematics[34]To understand the students about the geometric transformation, and put forward the corresponding teaching strategies according to the students understanding difficulties: understanding the nature of geometric transformation, introducing transformation from the things students are familiar with, and strengthening students meta-learning ability. Only from the perspective of investigation and research to study, but the idea of geometric transformation is not really integrated into the actual classroom teaching.

Chen Rongrong studied teachers understanding of geometric transformation through interviews, and teachers understanding of geometric transformation is relatively simple. Teachers at different levels differ greatly in their understanding of geometric transformation, and their consciousness of penetrating the idea of geometric transformation is also different[35]Therefore, it is necessary to improve teachers understanding of geometric transformation and their awareness of penetration in teaching.

For the seventh and eighth grade in Shanghai, Li Li conducted the teaching research on the infiltration of geometric transformation thought, and obtained the teaching results of geometric transformation infiltration through the analysis of students works[36]. The curriculum standard emphasizes that the teaching evaluation should be diversified, so the analysis of the teaching effect should be carried out in many aspects.

Throughout the development process of the teaching and research of geometric transformation thought, from the initial theoretical research to the present empirical research, the research of geometric transformation thought is constantly developing. The research should be based on the teaching practice, and the evaluation criteria of the results of the empirical research should be more diversified. Therefore, this research designs the quasi-experimental research on the idea of infiltration geometric transformation, and analyzes the infiltration effect from multiple angles after the experiment.

2.3.3 Related research on geometric transformation abroad

According to the ICMI study[37]As a result, many countries in the world attach great importance to transformation and symmetry in geometry courses. NCTM "School Mathematics Principles and Standards" requirements, from preschool to grade 12, all students should be able to use transformation, symmetry and other methods to analyze the mathematical situation. Therefore, for mathematics teachers, it is crucial to master the concept of transformation, so as to create a learning situation conducive to the development of mathematical thought. On this basis, the National Curriculum Center proposed the theoretical framework of learning advancement of geometric transformation[38], In order to evaluate the learning level of students geometric transformation, to provide theoretical reference for curriculum evaluation and teaching reform.

In in UK[39]In the combination of moment transformation, transformation and matrix representation is introduced, many places used geometric transformation, such as folding method for equilateral triangle, with rotation transformation Pythagorean theorem, and introduces the invariance in the transformation, the plane circle in the string rotation trajectory can produce envelope circle, space hyperboloid online segment of rotation, the design of the famous Chinese building small waist is the typical application of space segment rotation, it embodies the important application of geometric transformation in architecture.

By carrying out the exploratory teaching intervention experiment in the university classroom, Fan Lianghuo [40] discusses whether the application of geometric transformation is helpful to improve the students ability to construct auxiliary lines when solving geometric proof questions. The study focuses on high cognitive level or challenging geometric proof problems, and the results show that the transformation method can improve students ability to add AIDS to high cognitive level geometric problems, thus improving their geometric learning ability. This experiment provides some enlightenment for the infiltration practice study of geometry transformation in geometry classroom.

Edwards Based on the dynamic geometric environment, the students exploration and discovery of reflex composition are studied[41], Research has found that informal generalization based on a microscopic perspective helps students formal proof of learning. Guven studied dynamic geometry software in improving students understanding of geometric transformations[42], Research shows that dynamic environment can improve students understanding of geometric transformation to some extent because of its intuition and easy to operate. Yanik conducted an experimental study on the understanding of geometric transformations by pre-working primary school teachers in 2009[43]This study shows that primary school teachers understand the transformation through three processes: (1) the transformation as the ordinary movement of an object; (2) the transformation as the defined motion of an object; and (3) the transformation as the movement of all points on the plane.

2.3.4 Lack of the existing studies

By combing the existing research on geometric transformation, the research of geometric transformation is summarized into the following aspects: the instrumental role of geometric transformation, the application of geometric transformation in problem solving, the educational value of geometric transformation, the investigation and study of geometric transformation, and the penetration of geometric transformation. Existing research generally focus on from the perspective of theory to analyze the necessity of geometric transformation thought penetration, generally from the translation, rotation, axisymmetric transformation of the three contract transformation to study, with geometric transformation to recognize the triangle, add geometric transformation, and the development of the geometric transformation in the course. In addition, there are very few classroom experimental studies carried out at present, some of which are conducted in grade 7 and grade 8, and there is no research on understanding similar triangles from the perspective of geometric transformation, and there is no research on exploring geometric transformation relations in teaching to expand students understanding of transformation. So this study will try to geometric transformation in ninth grade geometry teaching teaching experiment, teaching experiment through the questionnaire, quantitative analysis and test results comparison and the combination of qualitative analysis, to fully reflect the teaching effect of infiltration geometric transformation thought, this is also an innovation in this paper.

2.4 Relevant theoretical basis

2.4.1 Van Hills theory of geometric thinking

On the study of geometric thinking, PierrevanHiele (PierrevanHiele, DinavanHiele)[44]The most famous is its research. Its geometric thinking level is divided into 5 levels or 3 levels, which is generally considered to be more accurate.

Level 1: Visualization. Through appearance recognition, geometric figure operation, students psychologically express these graphics as intuitive images, but can not understand the nature of the graphics, the basis of reasoning is intuition.

Level 2: Description / Analysis. Students use the nature of the graphics and understand the graphics and determine their characteristics. Through the operation to establish the nature, can classify the graph, can not understand the relationship of the graph. According to the classification of the graph reasoning, trying to establish the relationship of the graph.

Level 3: abstract / association. Can form an abstract definition, can understand the logical argument of geometry, and can carry out unformal argument, according to the nature of graph classification, form the relationship of graphic nature, reorganize the thinking. Level 4: Formal reasoning. Students establish theorems based on the system of axioms. Can carry out formal reasoning, reason the object is the relationship of graphic classification property, and express the relationship with logical chain in the geometric system.

Level 5: rigidity. Even without reference, students can also carry out formal reasoning in the mathematical system. The object of reasoning is the formal construction relationship, establish the connection of the geometric axiom system, and realize the formal operation. Students from a thinking level to the next higher thinking level of development is not achieved overnight, but according to a certain order of dynamic development. It requires the teachers to fully realize the students existing geometric thinking level, and effectively guide the classroom teaching. According to the existing thinking level of students, the corresponding teaching is carried out on the existing thinking level. This study, according to the geometric thinking level, designs the teaching case of geometric transformation thinking infiltration and carries out teaching practice.

2.4.2 Theory of sound sound thinking

From the perspective of cognitive psychology, thinking is a relatively complex internal operation that cannot be directly observed. Therefore, for the study of slow and implicit processing process like thinking, it usually adopts the method of vocal thinking, that is, using external language to manifest, so as to directly observe the whole thinking process. In 1945, psychologist Duncker.K first put forward the "method of vocal thinking"[45]However, it was not until 1972 that Newell and Simon successfully used this method in the field of problem solving, so far that the "sound thinking method" attracted the attention of researchers. The usual steps are as follows:

(1) For the advance training of the subject of sound thinking, the purpose is to be able to make sound thinking;

(2) Give the subject a homework, required to use the "sound thinking method" to complete, and use the recording equipment

Record the whole process of the subjects dictation;

(3) When the subjects vocal thinking pause, the experimenter reminds or asks him what he is thinking. Excluding special purposes, generally in order to not disturb the subject, do not ask questions when thinking;

(4) Organize the recorded language reports into written materials, and analyze the records to extract valuable materials, as a method to analyze the thinking process and thinking characteristics of the subject person. General teaching evaluation with test scores to evaluate students ability, such evaluation way cannot fully reflect the students to problem solving thinking condition, to deeply study the students thinking process, this paper to "loud thinking" in cognitive psychology for the study of the theoretical basis, mainly use language to show the students thinking, analyze the students thinking process, way of thinking and thinking ability, etc. This has not been seen in previous studies, which is also the innovation point of this study.

Adventitia section

Investigation on the teaching status of university geometry transformation

This chapter is divided into two parts, the first part includes the purpose, the survey object, questionnaire and the test volume, the second part includes questionnaire results statistical analysis and qualitative analysis of test results, through the investigation and test, understand the teachers and students to the understanding and application of geometric transformation, and preliminary analysis of the causes of geometric transformation teaching, lay a foundation for subsequent research.

3.1 Investigation Purpose and investigation object

3.1.1 Investigation Purpose

Educational research should be based on the teaching practice. In order to understand the current situation of geometric teaching in universities and universities, especially the specific teaching and learning situation of geometric transformation, we should compile a teacher questionnaire to investigate teachers understanding of geometric transformation and the penetration of geometric transformation in teaching. In order to understand the understanding of geometric transformation and the application of geometric transformation in geometric learning, the student questionnaire and the pre-experimental geometric test paper were prepared. The questionnaire focuses on non-intellectual factors such as the love for geometry, understanding of geometric transformation, and the use of geometric transformation in geometry learning, and the awareness of geometric inquiry and the ability to see problems from multiple angles.

3.1.2 Survey objects

In this study, all the mathematics teachers of a university and a university in Qingpu District of Shanghai were taken as the questionnaire survey. In the first mathematics teaching and research activity of the semester, 33 mathematics teachers were surveyed. After sorting, all were effective and further statistical analysis could be made. In this study, 212 students of grade 9 of the school were selected as the subjects of student questionnaire survey. On September 11, September 12020, a total of 212 questionnaires were distributed and 200 valid questionnaires were collected. The effective rate of the questionnaire was 94.33%. According to the analysis of the opening test results and the similar unit test results, 70 students from two parallel classes of grade 9 and grade 1 were selected. Using the 40-minute math class, the geometry test took the geometry test before the experiment. The math teacher invigilated and collected the papers, and 70 papers were distributed and collected.

3.2 Questionnaire preparation and pretest volume preparation

The teacher questionnaire took the form of a Likert Level 4 scale. The content includes two parts. The first part is the basic information of teachers: gender, title, teaching age, education, and grade; the second part is the understanding of geometric transformation and the infiltration of geometric transformation ideas.

Teacher questionnaire refers to the questionnaire survey on teachers understanding of geometric transformation in the existing research. Because this study also needs to understand the specific penetration of teachers in teaching, and other problems are also involved. The questionnaire is divided into four dimensions: dimension 1: teachers understanding of geometric transformation and thoughts, corresponding to the questions 1,2,14 and 16 in the teachers questionnaire; dimension 2:

the familiarity of questions 3,4,7 and 17; dimension 3: the attitude of questions 5,6,18,12,13,15,15 and 18; dimension 4: the penetration and application of geometric transformation ideas in teaching, corresponding to questions 8,9,10,1 and 11 in the teachers questionnaire. After the questionnaire was compiled, the survey was conducted, the questionnaire results were converted into excel data, and the excel data were tested for reliability and validity by SPSS25. From Table 3-1, the α coefficient was 0.861, the internal consistency of the scale was good and had good reliability. The validity analysis data show that the common factor variance of question 18 is 0.205 <0.4, which can be deleted. The deleted KMO value is 0.823> 0.7 shows that the validity is good, and the data analysis comparison factor is the expected one. See Appendix 1 for the teacher questionnaire.

Table 3-1 The Cronbach reliability of the faculty member questionnaire

number of entry	sample number	Cronbachα Coefficient
17	33	0.861

Table 3-2 Test of KMO and Bartlett

	KMO price	0.823
Bartlett Sphelicity test	Approximate chi square	568.044
	df	120
	P price	0.000

The student questionnaire includes non-intellectual factors such as students attitude to the idea of geometric transformation and the value of geometric transformation, as well as students application of geometric transformation thought methods in geometry learning. To student questionnaire preparation first choose a class to pretest, the results of the pretest into data with SPSS project analysis, project analysis results showed that 13,14 and total table correlation, can be deleted, delete the questionnaire reliability and validity are greater than 0.7, that the reliability is good, the student questionnaire is detailed in appendix 2.

Table 3-3 The reliability and validity analysis of the student questionnaire

number of	sample	Cronbachα Coefficient	KMO price

entry	number		
12	200	0.824	0.754

In order to improve the validity of the questionnaire survey and understand the use of geometric transformation by grade 9 students in the actual geometric problems, this paper compiled the geometric test papers before and after the experiment based on the existing masters and doctoral papers and the geometric questions in the textbook. Geometry before test paper including five geometric topic, test paper includes three geometric topic, 1,2 is the same topic, but the topic of examination Angle is different, geometric before test paper focus on whether students will choose geometric transformation problem solving, geometric test volume focus on whether students use geometric transformation for a problem solution and promote to explore. Test volume test translation, rotation, axisymmetric, and on the basis of the original topic requirements, using geometric transformation put forward new conclusions, geometric, test volume by the instructor and ninth grade teachers review, teachers give some changes, the corresponding modification after the final geometric test volume, geometric test volume see details in appendix 3.

3.3 Statistical analysis of the questionnaire survey results

Table 3-4 Basic information of the teachers

base situation		number of people	scale
sex	man	17	51.5%
	woman	16	48.5%
professional ranks and titles	elementary	14	42.4%
	middle rank	12	36.4%
	senior	7	21.2%

of school age	(0,6] Year	5	15.2%
	(6,15] years	12	36.4%
	(15,40] years	16	48.4%
record of formal schooling	undergraduate course	30	90.9%
	Master	3	9.1%%
Teaching grade	Sixth Form	7	21.2%
	Grade Seven	8	24.2%
	Eighth grade	8	24.2%
	9th Grade	10	30.4%

3.3.1 Teachers understanding and penetration of geometric transformation

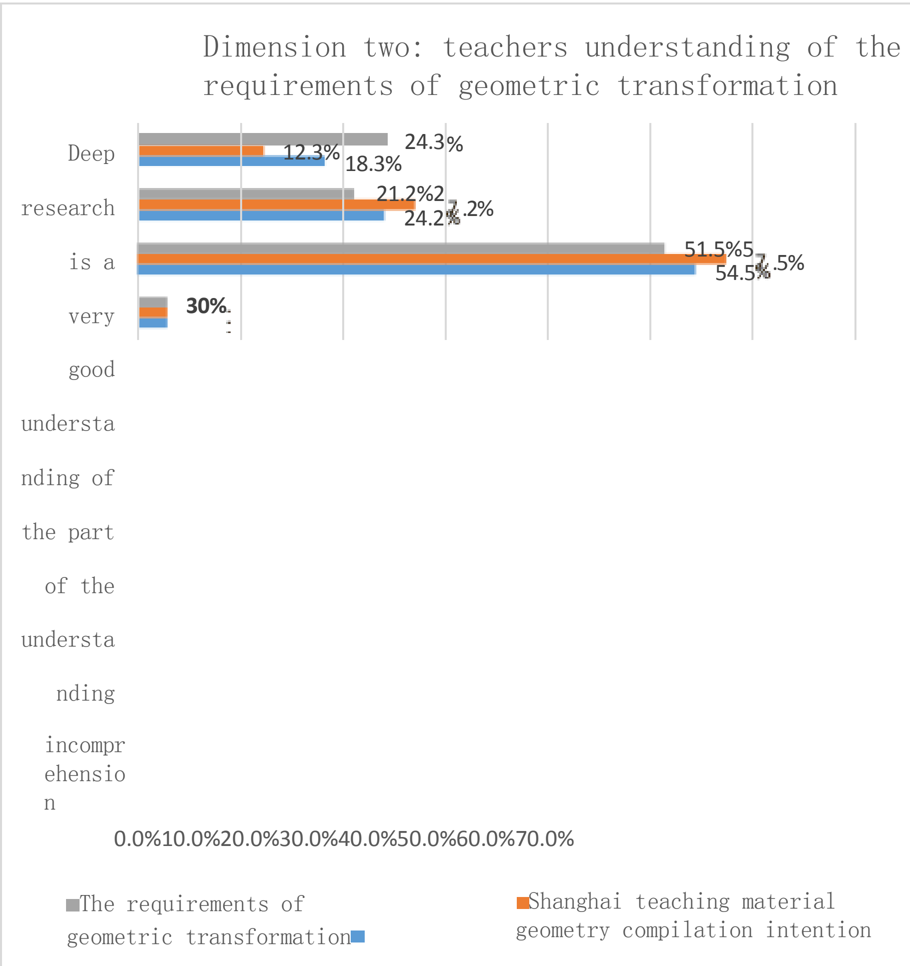


Figure 3-1 Teachers understanding of geometric transformation

According to the survey results, 54.5% of teachers only have a partial understanding of the requirements of geometric transformation in the Curriculum Standard, 24.2% understand them very well, and only 18.3% have studied geometric transformation in depth. Therefore, it shows that the depth of teachers understanding of the requirements of geometric transformation in the curriculum Standard needs to be improved.57.5% of teachers only have a partial understanding of the compilation intention of the geometric part of Shanghai textbooks, while only 27.3% are very understanding. Only 12.3% of teachers have conducted in-depth study on the compilation intention of the textbook, and they need further study on the compilation intention of the textbook. Only 24.3% of the teachers to penetrate the content of the geometric transformation thought in the thorough study, 21.2% of the teachers know very well, 51.5% of teachers knowledge understanding, nearly half of the teachers need to study teaching materials, mining materials can permeate the geometric transformation thought content, realize the value of geometric transformation in geometric teaching, attach importance to the geometric transformation in teaching penetration and application.

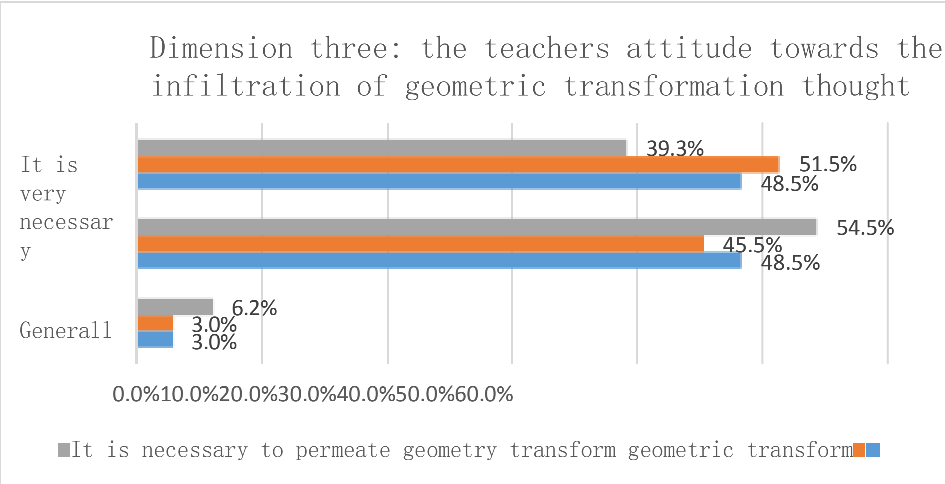


Figure 3-2 Teachers attitude towards the infiltration of geometric transformation ideas

Through statistical analysis, it is found that 51.5% of teachers think that the infiltration of geometric transformation thought is very necessary, and 45.5% of teachers think that it is generally necessary to infiltrate geometric transformation in teaching, which shows that teachers have a positive attitude towards the infiltration of geometric thought.48.5% of teachers believe that geometric transformation is very necessary for students learning, and 48.5% believe that geometric transformation is generally necessary for students learning. Generally speaking, teachers believe that geometric transformation is necessary for learning. The potential reason lies in that the geometric transformation often appears as the final question of the high school entrance examination, which is the difficult point that needs to break through in teachers teaching. In short, teachers can hold a positive attitude towards the infiltration of geometric transformation ideas, which lays a foundation for the subsequent teaching experiments.

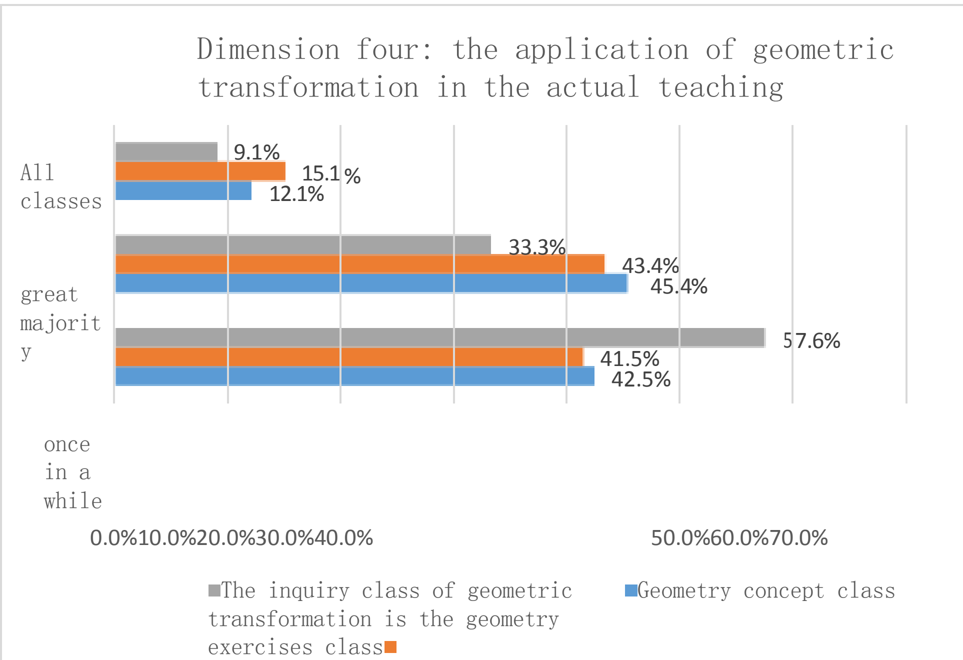


Figure 3-3 The application of geometric transformation in actual teaching

In the actual classroom, nearly half of the teachers only occasionally use the form of geometric transformation in concept classes and exercises, and only about 12% of the teachers who can pay attention to geometric transformation in all courses. It can be seen that in the actual teaching, teachers do not really pay attention to the application of geometric transformation.

Practice in the process of using the opportunity to lecture geometry class classroom observation, observed that the teacher in the geometry classroom is not a lot of use geometric transformation, which in seven and eighth grade teachers only in the import process of geometry transformation, ninth grade teachers in teaching rarely use geometric transformation to guide students to geometric problem solving, only when the geometric problems to use geometric transformation to guide. Most of the teaching methods used by teachers are the combination of teaching and practice, and the modeling of geometric questions. Students only need to remember the corresponding model, and students can master the relevant content by imitating the teachers teaching method. The teachers explanation of the geometric questions is mostly the specific requirements to complete the questions, as long as the students complete the requirements of the questions, many questions did not dig deeply from the perspective of geometric transformation. Teachers should have the knowledge basis of geometric transformation and the idea of geometric transformation, with the purpose of developing students core literacy, and university geometry teaching should try to permeate the idea of geometric transformation.

3.3.2 Students understanding and application of geometric transformation

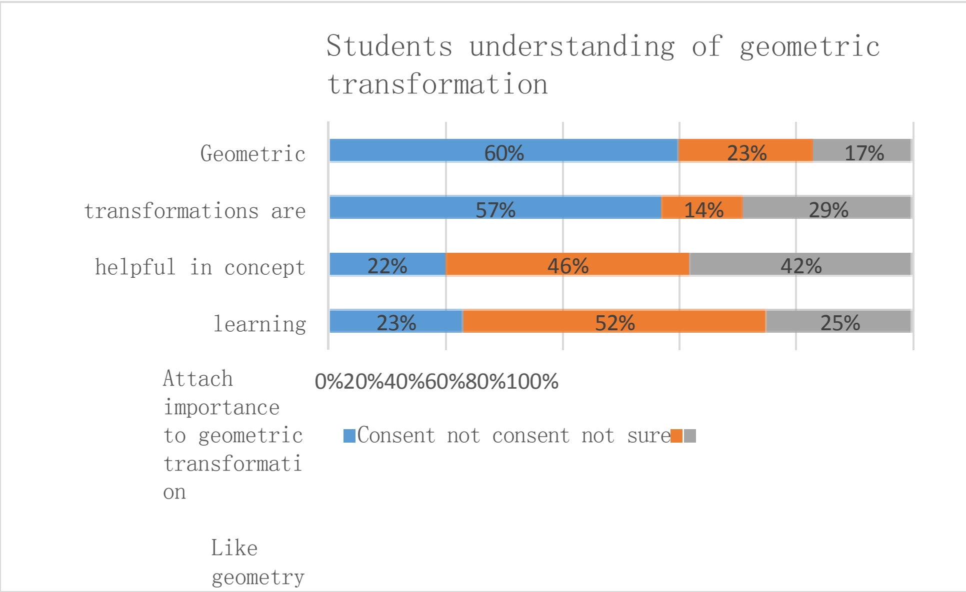
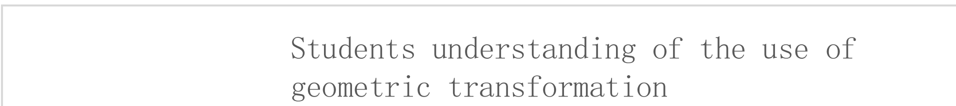


Figure 3-4 Students understanding of geometric transformation

Questionnaire survey statistics show that about 60% of students think geometric transformation contributes to the concept of learning and geometric problem solving, but only about 20% of students like geometry and geometric transformation, which to some extent students on the value of geometric transformation is a positive attitude, because of various reasons, teachers in teaching often does not emphasize the importance of geometric transformation, so the students will ignore the value of geometric transformation, which requires the teacher in the teaching emphasizes the importance of geometric transformation and its application.



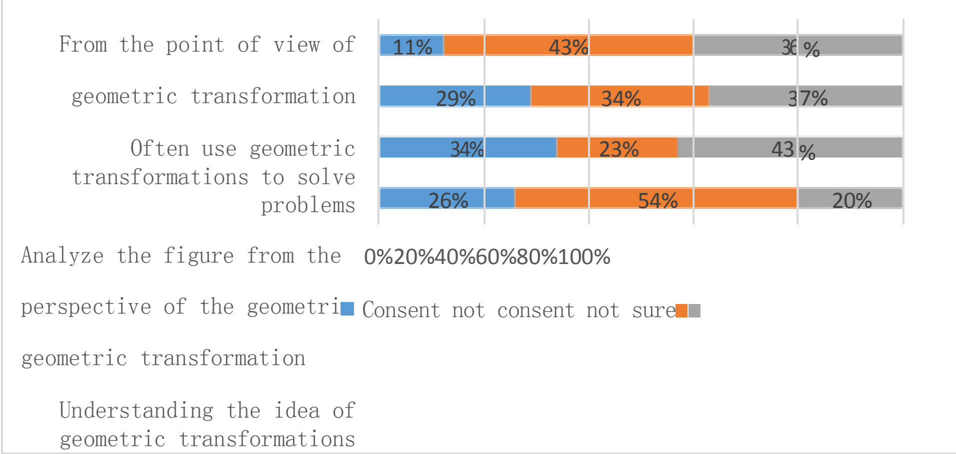


Figure 3-5 Students understanding of the use of geometric transformation

Questionnaire survey statistics show that only about 30% of the students understand the idea of geometric transformation, from the perspective of geometric transformation analysis graphics, with geometric transformation, using the geometric transformation of student proportion is not high, only 11% of the students from the perspective of geometric transformation, further illustrate the necessity of penetration of geometric transformation thought. To further validate the situation reflected in the questionnaire, a qualitative analysis of the geometric test results was performed.

3.4 Analysis of students test results

1. The four vertices of the square CDEF are located on the edge of △ ABC, AE = 5, EB=7, then the sum of the areas of △ ADE and △ BEF is.(Write out the solution process)

This question examines the triangle similar content, the question type is filling in the blanks, in order to examine the method used by the students, the requirement to write the answer process. The conventional solution is easy to think of, but it is very large, students are prone to error when calculation, the second optional method is rotating, without the geometric transformation consciousness of this method is not easy to think of, the advantage of this method is simple, and discussion with the instructor, this question can be used as the students in geometric transformation application test. By correcting the examination paper, the students problem-solving methods are counted. The specific situation is as follows:

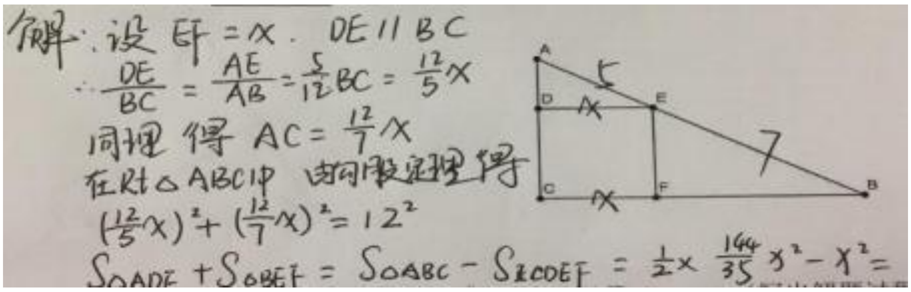


Figure 3-6 Example of misanswer in question 1

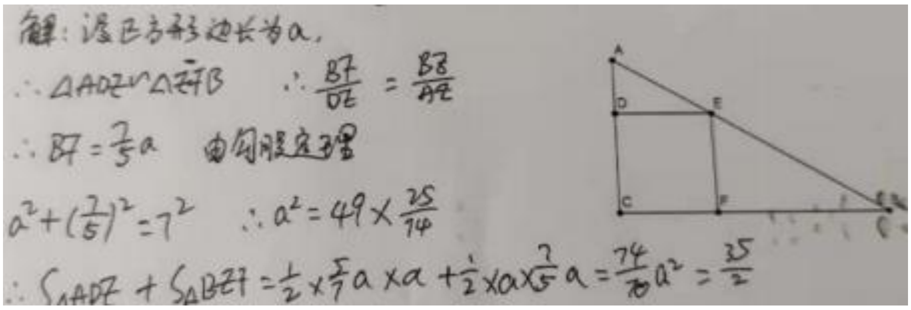


Figure 3-7 A correct answer to question 1

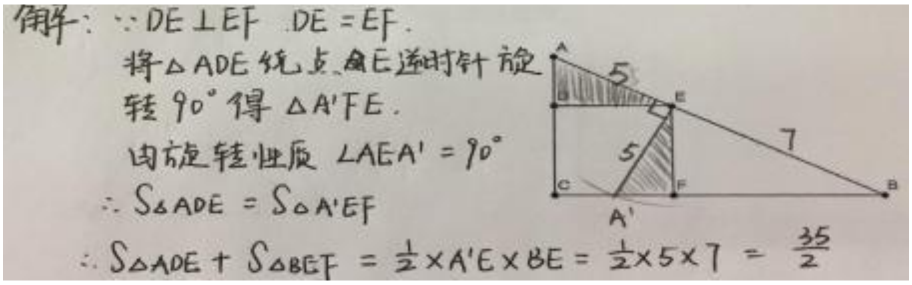


Figure 3-8 Example of geometric transformation in question 1

60% of the students use similar or parallel lines, and then use the Pythagorean theorem to calculate, but do not get the correct answer due to the calculation error (as shown in Figure 3-7). Thirty percent of the students used similar or parallel methods, and then got the correct answer (as shown in Figure 3-8). Only 10% of the students used the rotation to rotate two separate triangles together, using the nature of the rotation to quickly get the correct answer (as shown in Figure 3-9).

2. Figure Figure, in a square grid with side length 1, find ABC + ACD =. (Write the process of solving the problem) This question comes from a question on the similar triangle exercise book. By changing the square grid to more, the solution of the questions also becomes diversified. The conventional solution of this question is to use similar triangles to achieve the transformation of the Angle.

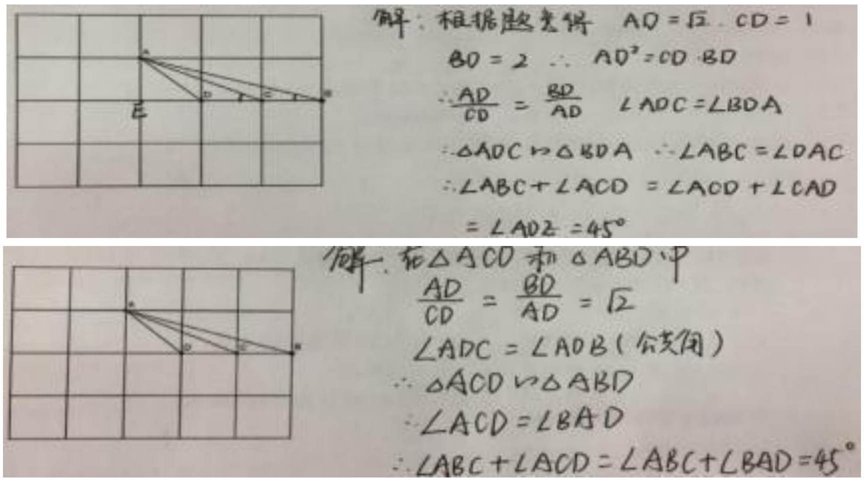


Figure 3-9 Similar Solution Code Example in Question 2

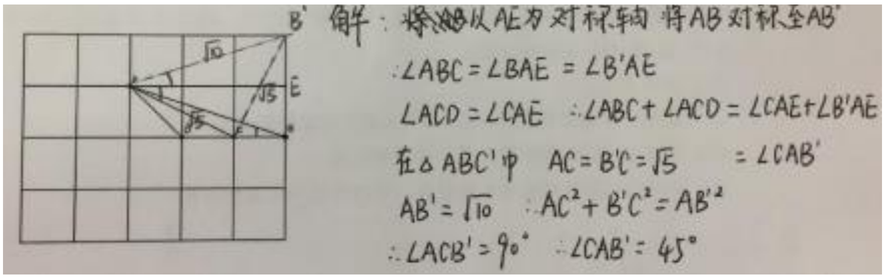


Figure 3-10 Example of geometric transformation solution in problem 2

92% of the students chose to use a similar triangle to achieve the position of the angle, thus finding the degree of the angle. Only individual students will use similar triangles, and then use translation, axial symmetry and other methods to achieve the transformation of the position of the Angle, and then find the degree of the Angle. Through the analysis of the students test papers, most students do not have the consciousness of thinking about the problem from the perspective of geometric transformation. Even though the method of geometric transformation can make the problem solving simple and fast, most students still use the complex solution method. Students geometric problem solving is only satisfied with the requirements of the problem, students idea of solving problems is single, in the process of solving the problem, they can not try multiple solutions, the application of geometric transformation in geometric problem solving is not optimistic, this phenomenon during the internship homework correction and students math class performance has also been confirmed.

The third question examines the students understanding of the essence of the auxiliary line. The students do the questions as follows:

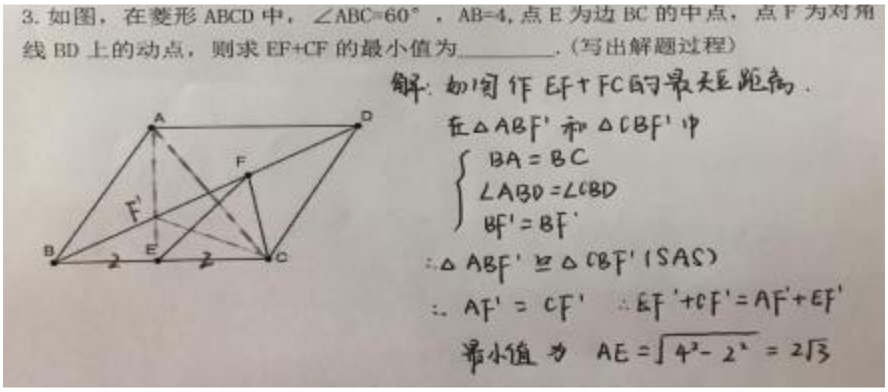


Figure 3-11 Question 3

Most students are from the perspective of equality to argument AF=CF, few students from the perspective of axisymmetry, students to add the essence of the auxiliary line from point A and C about straight BD symmetry, most students union AF, but do not understand the nature of the auxiliary line is produced by axisymmetry, visible students in A passive learning state, only know so, dont know why to do, in the usual learning not with "why" to learn, such geometric learning can only be mechanical and inefficient.

Question 4 is examined from the perspective of geometric transformation to explore the new conclusion, students test situation is presents the phenomenon of polarization, poor basic requirements of the students, learning ability of students can not only correct problem solving, even can get some new conclusion, but can explore the students are few, students specific problem solving is as follows:

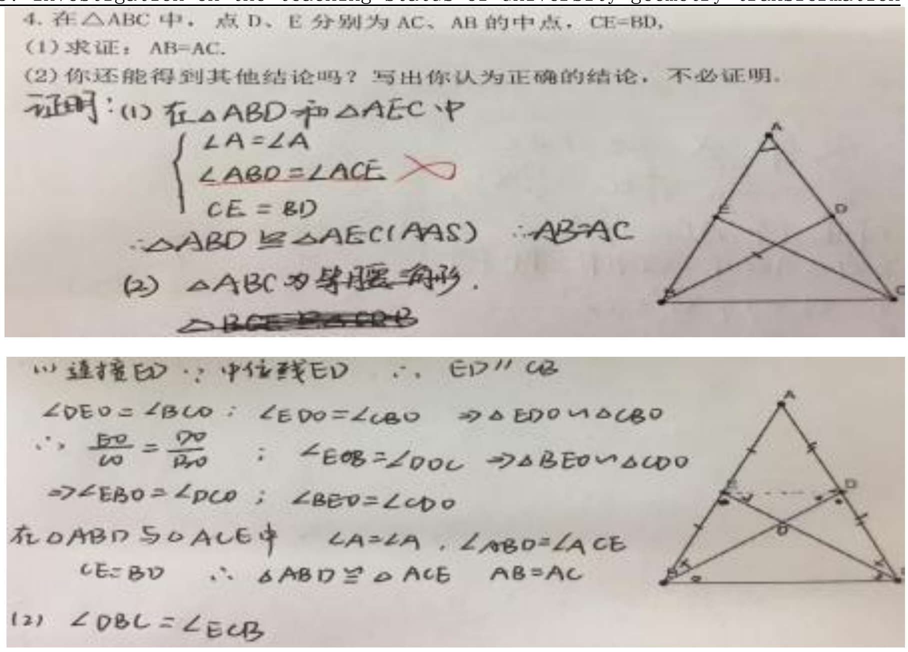


Figure 3-12 Question 4

The fifth question in the geometric test paper before the experiment aims to test whether the students can consider the problem from the perspective of geometric transformation and assist the geometric proof from the perspective of motion transformation. The first question examines the basic knowledge, students can generally solve the problem through the proof of equal triangle; the second question examines the geometric concept of motion change, students can get △ CED by rotation △ ABD under the prompt, but the students do not fully explain the three elements of rotation: rotation center, rotation Angle and rotation direction. Third, the students all use the CEAB; fourth, the students write other conclusions are various, lack of logic. The specific answers are provided as follows:

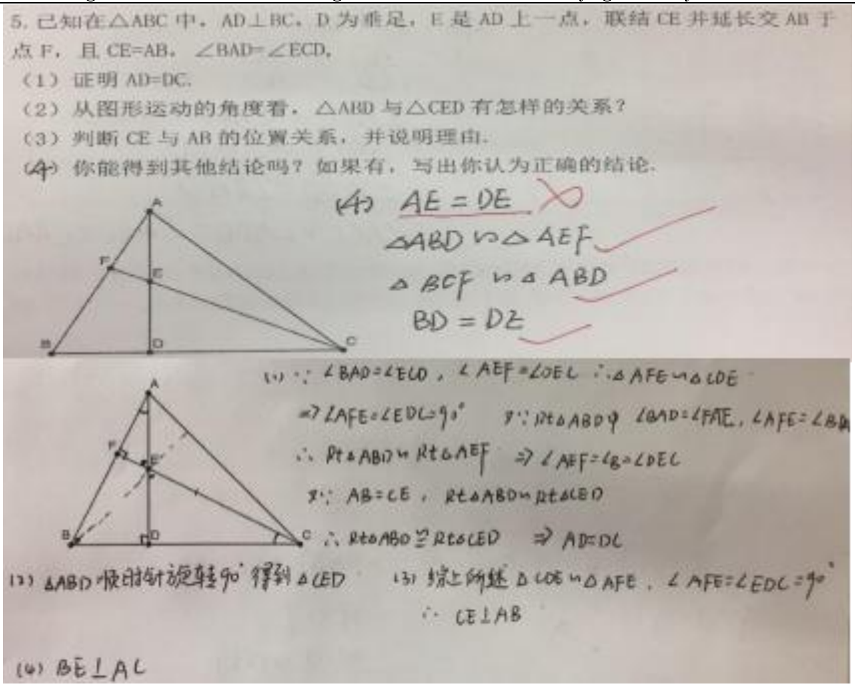


Figure 3-13

For line segment, Angle equal method, students in the process of geometric problem solving first think of is equal, similar methods, and geometric transformation method using consciousness is not strong, geometric exploration ability to improve, it more illustrates the geometric transformation in students learning is not get the attention of students and teachers, and graphic movement related topic students also have difficulty.

3.5 Reason analysis of the teaching status of geometric transformation

3.5.1 Teachers do not pay enough attention to the application of geometric transformation ideas

In teaching, teachers understanding of geometric transformation and the concept of geometric teaching directly affect students geometry learning. The new curriculum reform has been transformed from the traditional "double base" requirements to the current "four base" requirements. Whether the new basic ideas and basic activity experience can be truly implemented in the geometry teaching lies in the teachers understanding and implementation of the new curriculum reform concept. Due to the limitation of the number of class hours and the capacity requirements of content, grade 9 teachers still adopt the traditional teaching mode combining teaching and practice in teaching. What teachers generally pursue the firmness of students knowledge in class, and often fail to really realize the importance of geometric transformation to geometric learning and ignore the penetration of ideological methods. In geometry teaching, teachers require students to remember geometric models, whether students really understand geometric models, understand the connection between geometric models, and understand the conditions for the use of geometric models. Finally, they can only remember and consolidate through "brushing questions". Whether geometry teaching really improves students geometry ideas and develops students inquiry and innovation ability, teachers still lack corresponding attention. Therefore, from the overall perspective of students development, teachers should update the concept of geometry teaching. To change the current geometry teaching mode and deepen students understanding of geometry, teachers still need to deeply understand the goal of geometry teachingUniversity geometry teaching is not only a carrier to cultivate students logical reasoning ability, but also a way to help students build a broader subject vision

3.5.2 Students concept of movement and transformation needs to be improved

Due to the high content and high requirements of mathematics learning in college, most college students are in a passive state of learning geometry to cope with exams, so they are not very interested in learning geometry and lack the consciousness of inquiry and innovation in the process of learning geometry. When correcting the ninth grade math homework, I found that students lack flexibility in the process of geometry problem solving, using translation, rotation and axisymmetry to solve geometric problems. Students only discuss the topic, only focus on whether the answer to the question is correct or not, and the idea of geometric transformation is not enough.

Throughout the current situation of university geometry teaching and learning, from the current teaching results, the existing geometry teaching and learning mode is fruitful, but it is not conducive to the long-term development of students. And penetration of geometry transformation thought of university geometry teaching, not only can change the students cognition of geometry learning, and for high school mathematics learning matrix and transform the ideological foundation, even for the theory of the seeds of thought, infiltration geometry transformation thought geometry teaching is beneficial to the future development of students.

Adventitia section

Teaching analysis of geometric transformation thought infiltration

In chapter 2 research review and chapter 3 university geometry transformation teaching status survey, on the basis of the main content of this chapter is the penetration of the teaching material carrier, and combined with the existing research method of the teaching principle, the penetration of geometric transformation thought principle and build knowledge and thought double-level teaching goal, according to the students geometric thinking level design teaching case.

In the teaching experiment conducted in a university in Qingpu District, Shanghai, we should first clarify the requirements for geometric transformation in Shanghai Mathematics Curriculum Standards for Primary and Secondary Schools, and sort out and dig out the content carriers through which the idea of geometric transformation can be permeated. The compilation of geometric content in the Shanghai edition university textbook provides convenience for the infiltration of geometric transformation thought. Through the infiltration of geometric transformation thought, it helps students to build geometric knowledge system, so that students can absorb the idea of geometric transformation, form the concept of geometric transformation, and apply it to geometry problem solving. How to effectively infiltrate the idea of geometric transformation into the geometry teaching is the problem to be studied in this paper. Before the teaching design, teachers should pay attention to fully explore the permeable geometric concepts, theorems, exercises, graphics, and comprehensive practical activities.

4.1 The penetration carrier of the geometric transformation thought in the textbook

After combing, it is found that there are not only a large number of geometric concept carriers that can penetrate the thought of geometric transformation in the textbook, but also a large number of examples and exercises that can penetrate the thought of geometric transformation in the textbook. Moreover, there are some geometric figures in the textbook that appear repeatedly in the textbook, and the only change is the conditions and conclusions of the topic. If only in the teaching on the topic, is bound to increase the burden of students, and even make students feel difficult to grasp the geometric topic, so that students have "fear of difficulties" mood. If the teacher can make full use of the typical graphics in the textbook, guide the students to explore the connection of the topic of the same figure from the perspective of geometric transformation, encourage the students to summarize, let the students try to compile the questions from the perspective of the author. In this way, we can not only change students habit of learning, but also cultivate students to grasp the invariable nature in the process of geometric figure transformation, sort out the knowledge points in the textbook, and the carriers are as follows:

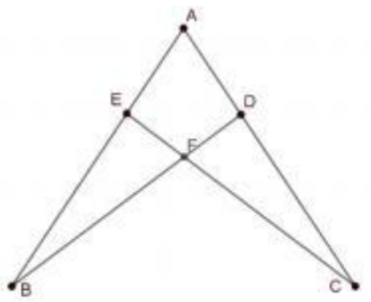


Figure 4-1 Typical axisymmetric figures in the textbook

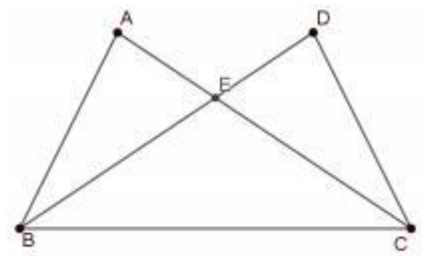


Figure 4-2 Typical axisymmetric figures in the textbook

Table 4-1 The carrier of knowledge penetration in the teaching textbook

translation transformation	Chapter 7 Comparison of line segments and angles: overlap Chapter 11 Design of motion plane graphs Chapter 13 parallel line drawing P51 Chapter 14 Operational experiment and proof of the triangle P79 The concept of an equivalent triangle P86 Equal triangle determination theorem SAS, ASA reasoning P92
rotation transformation	Chapter 4: Front picture, the concept of circle, the concept of circle center corner Chapter 7 Front diagram, concept of Angle P92, residual corner of the same angle (complement) equal P104 (extensible) Intuitive understanding of the three positional relations of two straight lines in space P116 Chapter 11 Design of plane graphics P111, design of plane Mosaic figures P112 Chapter 13 The concept of equal top angle P39, vertical line segment P42 Chapter 14 Operational experiment and proof of the triangle P79 The outer angle of the triangle and 360° sense P83, the concept of the complete triangle P86 Equal triangle determination theorem SAS, ASA reasoning P92 Chapter 19 Right triangle diagonal edge midline is equal to half of the oblique edge proof (times the length of the midline nature) P115 Chapter 22 Polygon Diplomacy and intuitive interpretation P70, parallelogram center piled into graphics P73

Axial symmetry transformation	Chapter 7 Generation process of the angular bisector P100 Chapter 11 Design of plane graphics P111 Chapter 13 Line Section Vertical bisector Line P43 Chapter 14 Operation of the folding of the isosceles triangle and proof of the auxiliary line P104 Chapter 19 The proof of the determination theorem SSS P90 Import P102 of vertical bisector of line segment and import P103 of angular bisector

In teaching, we should make full use of these knowledge penetration carriers, deeply study the idea of geometric transformation behind these knowledge, and reasonably infiltrate the thought of geometric transformation into each link of teaching. In the process of feeling, understanding, mastering and flexibly using knowledge, students thoughts and consciousness of geometric transformation are also gradually enhanced.

Table 4-2 The exercise penetration carrier in the teaching textbook

Textbook chapter	rotation transformation	Axial symmetry transformation
Chapter 14	Example 1. Exercise 2. Example 2. Example 3.P95 Practice 3.P96 cases 5.P97 Exercise 2. Exercise 3.P98 cases 7. Case 8.P99 Exercise 1. Exercise 2.P100 cases 12.P102 Exercise 2. Exercise 3.P103 example. Exercise 1.2.3. (common graphics) P114	Example 4.P95 Example 6.P97 Example 9.P100 Exercise 1.P101 Exercise 1.P103 Example 2.P110

Chapter 19	Exercise 3.P92 cases 5.P96 Exercise 1.P93 Example 8. (available for further exploration) P94 Example 11. The nature of the times-long midline (Grade 9 can be extended to the bisbisproperty) Example 13.P99, Case 8.P131	Case 3.P90 case 4.P91 Exercise 1. Exercise 2.P92 cases 6.P93 Exercise 2.P03 cases 7.P93 Exercise 1.P94 Practice 2.P94 Cases 10.P96 Practice 2.P97 Exercise 2. Solve P98 more for one problem Case 2. Case 3.P107.1.P113

With the corresponding penetration carrier, the next step to consider is what principle should the idea of penetration geometric transformation follow? What kind of teaching objectives should be established in the teaching of infiltration geometric transformation thought? What effective measures are taken to permeate the teaching? Therefore, the relevant research of mathematical thought and method is sorted out, and the teaching principles of mathematical thought in the existing research, considering the particularity of geometric transformation infiltration, the corresponding infiltration principle is formulated, the teaching objective level is determined, and the corresponding teaching infiltration measures are summarized.

4.2 The principle of thought penetration of geometric transformation

As a method of mathematical thinking, geometric transformation should follow the teaching principle of general mathematical thinking method. Shen Wenxuan wrote in the Mathematical Thinking Methods in the University[46]It puts forward four teaching objectives and principles of mathematical thought methods, but the teaching of mathematical thought cannot be accomplished overnight. Therefore, the ideological infiltration should follow the principle of step by step, and the infiltration should also be based on the law of students cognitive development and fan Hills geometric thinking level, but also follow the principle of acceptability. Classroom teaching should be hierarchical, systematic and comprehensive analysis. This research puts forward the teaching objective principle of penetrating the idea of geometric transformation:

(1) The principle of goal-orientation

The thinking method should conform to the "curriculum standard", based on the development of students, in line with the reality of students, establish the level of geometric transformation thought in the teaching goal, and formulate the corresponding teaching objectives of the ideological level.

(2) The principle of turning the hidden into the explicit

The idea of geometric transformation is implicit, but the method of geometric transformation is explicit. The recessive idea is carried out in the teaching process through the explicit method, so as to achieve the teaching goal of thought, such as the idea of rotational transformation through the method of rotational transformation.

(3) The principle of integration and unity

The thought goal is combined with the knowledge goal, and the teaching goal of the mathematical thought method is combined with other cognitive goals.

(4) Principles of science and practicality

The teaching objectives of formulating mathematical thinking methods should be accurate, clear, in line with the reality, with a relatively appropriate horizontal level and horizontal and horizontal network level.

(5) The principle of gradual progress

The idea of penetrating geometric transformation is a long-term task. According to the cognitive objectives stipulated in the Curriculum Standards, the idea of penetrating geometric transformation can be divided into four levels: perception, understanding, internalization and flexible application. For example, combining the idea of graphic motion perception; understand the idea of geometric transformation; internalize and flexibly apply the idea of geometric transformation, following the principle of spiral rise and gradual deepening.

(6) The principle of acceptability

According to the students cognitive level and fan Hills geometric thinking level, under the premise that students can accept, the geometric transformation thought to permeate, so that students can constantly understand and understand the transformation thought.

4.3 The teaching objective level of geometric transformation thought

On the basis of curriculum standard, teaching content and students cognitive development law, Dong Lei divides the level of teaching objectives of mathematical thought from two aspects: teachers teaching objectives and students learning objectives[47]. The teaching objectives are divided into "infiltration-visualization- -application"; the learning objectives are divided into "sense-perception-understanding-formation-mastery-application-internalization"; corresponding to the teaching objectives of "memory-interpretation-understanding-inquiry understanding", the above objectives are divided into low to high levels, and the structure of these target levels can be used as the theoretical basis for infiltration and application of geometric transformation ideas.

Table 4-3 The level of teaching objectives of university mathematics thinking methods

administrative levels	learning target	instructional objectives	Teaching objectives in the cognitive field
A	Feel, feel	osmosis	memory level
B	Understanding and formation	clear	Level of explanatory understanding
C	Master, apply, and internalize	utilize	Level of inquiry-based understanding

4.4 Teaching measures for the idea of penetrating geometric transformation

Wu hyperplasia believes that "the learning of mathematical thinking and methods has gone through four basic stages: imitation experience, clarity, application consolidation and connection development". From the basic stage, the basic strategy of developing the teaching method of "teaching" and "learning" is: firstly, including the mathematical thinking method in the interaction of "teaching" and "learning"; thirdly, consolidating the mathematical thinking method in the application and training of "teaching" and "learning"; and thirdly, developing the mathematical thinking method in the interaction of "teaching" and "learning"[48]。

In order to make the idea of geometric transformation permeate smoothly in the university geometry teaching, teachers should first grasp the students geometric thinking level scientifically and accurately. Combined with the actual geometric thinking level of students, this paper constructs a geometric teaching system that permeates the thought of geometric transformation, pays attention to the dual teaching objectives of knowledge and thought, and carries out proper penetration in each link of teaching. Previous studies have shown that the geometric thinking of grade 9 students is generally in level 2: description level and level 3: non-formal deductive level. Ninth grade students generally consider it too difficult when it comes to geometric transformation. In order to gradually reverse students attitude towards geometric transformation, design the activity to experience the idea of geometric transformation.

4.4.1 Graphic cutting and spelling experience the idea of geometric transformation

In teaching, origami activities and graphic cutting activities can be designed. In the operation and thinking activities of graphic cutting, on the one hand, students intuitive geometry should be cultivated; on the other hand, students geometric thinking level can be improved. Through the graphic cutting activity, students can not only experience the common graphic movement, but also experience the invariable ideas in the process of graph movement transformation.

Teaching knowledge objective: the movement of the triangle middle line and the graph

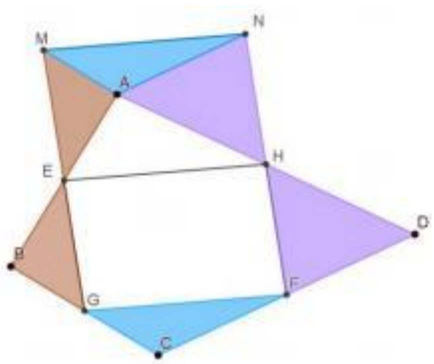
Teaching thought goal: the constant thought in the transformation of perception and perception

The tools needed for teaching are paper, scissors and ruler. Reviewing the nature of the parallelogram, the teacher should ask: how can we cut a piece of paper into a parallelogram? Use this question to cause the students to think about it. The original knowledge of students is that the line segment connecting the midpoint of any quadrilateral forms a parallelogram. Therefore, students will try to cut off the four corners of the corresponding quadrilateral and get a parallelogram (knowledge goal 1: the middle line of the triangle). Then the teacher seized the opportunity to continue to ask the second question: cut four corners can be spell into what graphics? The students kept trying and found that the remaining four pieces could be put together into a parallelogram.

The teacher asked the students to recall the process of cutting and stitching, and draw geometric figures. Question 3: What kinds of graphics movements are used in this process? In the comparison, the students found the rotation and translation motion (knowledge goal 2: graphic movement). As shown in Figure △ BEG around point E 180° to obtain △ AEM; △ DFH around point H 180° to obtain △ ANH; △ CGF translation along the ray CA (see Figure 4-3).

Through the graph movement, the students feel that the puzzle can change the position of the small triangle without changing the shape and size, which is the invariant idea in the process of the graph movement transformation. Through the drawing, students can also understand the essence of the long period, that is, the thought of rotation transformation (thought goal: feeling the constant thought in the transformation).

Students can continue to explore the question four: if still this quadrilateral, can only cut two knives into a parallelogram, how to cut the quadrilateral? Cut the puzzle along the line of the opposite point, as shown in Figure 4-4. The idea of invariant in the process of motion transformation is the idea of geometri ill also be used in the future learning. Studen y convex quadrangle cutting square after class.



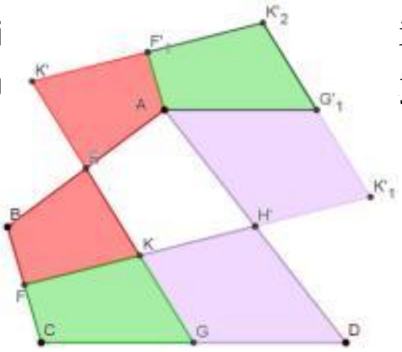
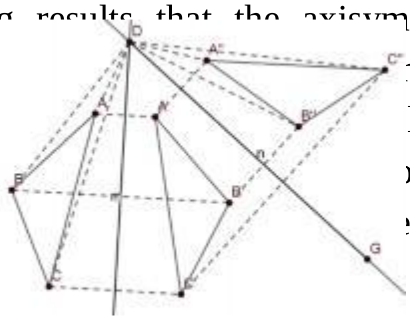


Figure 4-3 parallelogram Figure 4-4 parallelogram

4.4.2 Explore and understand the idea of geometric transformation

In order to understand the idea of geometric transformation, we should first have a deep understanding of the relationship between the three geometric transformations. The result of two translations is one translation, let the students think about what is the transformation of two axisymmetric synthesis? In teaching, students can explore through mapping, which can not only cultivate students geometric mapping ability, but also deepen their understanding of the most basic geometric transformation in the process of exploring the relationship between geometric transformation.

It can be seen from the drawing results that the axisymmetric transformation is the most basic ge… …nd the following conclusion is drawn: if the a… …l, then two axisymmetric times is equivalent … …own in Figure 4-3. If the axis of symmetry i… …s tries equivalent to one rotation, as shown in F…



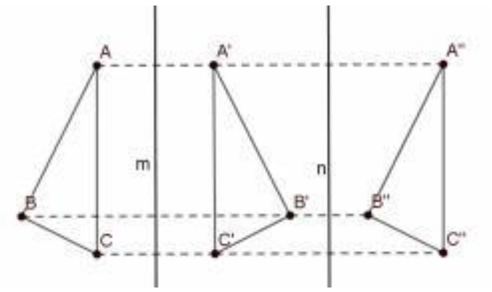


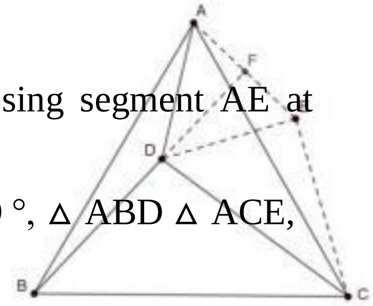
Figure 4-5 Two symmetry axes are parallel Figure 4-6 The symmetry axes intersect twice

4.4.3 Try to solve one problem more than one solution to grasp the idea of geometric transformation

example. As shown in Fig, point D is a point inside the equilateral △ ABC, DA=3, DB = 4, DC=5, and find the side length of △ ABC.

Solution method 1: (rotary transformation)

The △ ABD rotates the △ ACE by 60° counterclockwise centered on point A,



Link segment DE, extension segment BD crossing segment AE at point F.

Known from the rotational properties, DAE = 60 °, △ ABD △ ACE,

AE=AD=3,CE=BD=4；

So the △ ADE is an equilateral triangle,

So DE = 3, AED=60°;

In △ DCE, DE = 3, CE=4, CD=5,

DE2+CE2=32+42=52=CD2；

So the △ CDE is a right triangle, with DEC = 90 °;

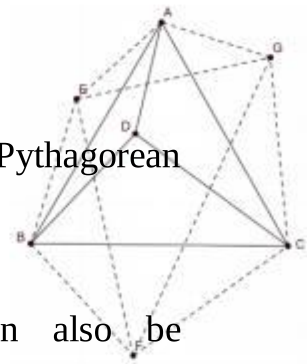
So AEC = AED ~~+ DEC =~~ 60 ° + 90°=150°; Figure 4-7 rotation method

So ADB = 150 °, ADF = 180 ° -ADB = 30 °,

DAE + ADF = 60 ° + 30°=90°, so DFAE;

In Rt △ ADF, AFD = 90 °, ADF = 30 °, AD=3,

So AF=, DF=3, BF=BD + DF=3,$\frac{3}{2}\frac{3}{2}\sqrt{}$ 4 + $\frac{3}{2}\sqrt{}$



In Rt △ ABF, AB2=BF2 + AF2 is obtained from the Pythagorean theorem

So you can get AB= 25 + 123;

Where other similar rotational transformations can also be performed.

Solution 2: (axisymmetric transformation)

Change △ ABD axisymmetrically with linear AB to the △ ABE;

With the straight line BC as the axis of symmetry, transform △ BCD as axisymmetrically to △ BCF;

Change the △ ACD to the △ ACG;

Link line segment EF, FG, EG;

According to the axisymmetric properties available, △ ABD △ ABE,

△BCD≌△BCF， △ACD≌△ACG；

AE=AD=AG=3； BE=BD=BF=4； CF=CD=CG=5；

∠EAG=2∠BAC=120°,∠EBF=2∠ABC=120°,

∠FDG=2∠BCA=120°,

So the △ AEG is an isosceles triangle, EG = 3 AE = 33;

△ BEF is an isosceles triangle, EF= 3 BE = 43; Figure 4-8 Axsymmetry

△ CFG is an isosceles triangle, FG = 3CF=53;

In the △ EFG, FG 2 = EF 2 + EG 2;

So the △ EFG is a right triangle, with a FEG = 90 °;

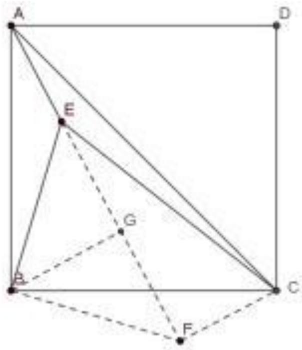
Because 2 SABC = SAEG + SBEF + SCFG + SEFG –

所以$2 \times \frac{\sqrt{3}}{4}$AB2=√3×4√3； $\frac{\sqrt{3}}{4} \times 9 + \frac{\sqrt{3}}{4} \times 16 + \frac{\sqrt{3}}{4} \times 25 + \frac{1}{2} \times 3$

So AB= 25 + 123;

Variant: as shown in the figure, a point in the point E square ABCD, EA=1, EB=2, EC=3, find the side length of the square ABCD.

Analysis: In the square, AB=BC, ABC=90°. The known three line segments are not in a triangle and can not be directly calculated directly. Therefore, we can consider rotating 90° to concentrate the three line segments in a graph, and therefore consider rotating △ ABE around point B 90° clockwise.



Solution method 1: (rotary transformation method)

Turn the △ ABE clockwise around point B for 90°

△ CBF, linked EF, over point B as BGEF at point

By the properties of rotation, △ ABE △ CBF,

BE=BF=2，AE=AF=1；

∠EBF=∠ABC=90°,EF=2√2，AE⊥CF；

In △ CEF, EF2 + CF2= (22) 2 + 1=32=EC2; Figure 4-9 rotation method

So △ CEF is a right triangle, and EFC = 90 °,

Therefore, AEB = BFC = 90 ° + 45°=135°;

So A, E, F and the three points are collinear.

B GAF, point G is the midpoint of EF, BG = EG = FG = 2, AG = 1 + 2;

In Rt △ ABG, AB 2 = BG 2 + AG 2 is obtained from the Pythagorean theorem;

AB2=5 + 22 that is, AB= 5 + 22;

Conclusion: Here, the dispersion conditions are concentrated through the rotation transformation, and the Angle of the corresponding line segments before and after the rotation transformation is the rotation Angle. In this way, you can easily understand AEFC, EFFC by the Pythagorean theorem, thus three collinear A, E, F, this method using the idea of rotation transformation, using rotation transformation can solve a kind of problem, known the equilateral triangle inside a distance between three vertices, for the long or Angle, known square inside the distance between three vertices problem (essentially isosceles right triangle inside a problem), rotating transformation can simplify complex problems.

Solution method 2: (axisymmetric transformation method)

The problem of side length is transformed into area problem, axisymmetric transformation and equal area method.

With the straight lines AB, BC, and AC axes, respectively,

Change △ ABE, △ BCE and △ AEC

△ABF、△BCG、△AHC，

As known from the axisymmetric properties

△ AFH and △ CHG are all isosceles right tr

△ FGH is a right triangle;

So 2 SABC = SAFH + SCGH + SFGH

Is the AB= 5 + 22;

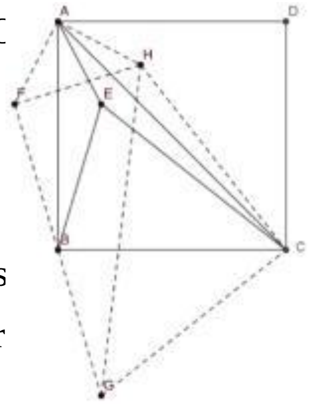


Figure 4-10 Axisymmetric method

Conclusion: This solution uses axisymmetric transformation, through axisymmetric transformation and equal area method, transforms a problem of line segment length into a problem of area finding, and obtains isosceles triangle through axisymmetric transformation. In comparison, the method of axisymmetric transformation is more general, can be applicable to isosceles right known EA, EB, EC in the triangle of any length, combined with geometric Helene formula and axisymmetric transformation, this method is applicable to isoscilateral triangle, isosceles acymetric triangle, for this problem axisymmetric transformation is more generally applicable.

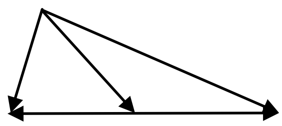
Find an orthogonal projection of points A (2,1,4), B, M (5,3,2), N, 3, -1,4) on 1 in a line.

amusement. AB (2,4,2) MN (2,1,2) introduces vector A and represents the length of the search projection by l. Then, using the average value, we obtain the value

$$l = \text{пр}\ MN\ AB = AB \cdot MN\ MN = 2 \cdot -2 + -4 \cdot -1 + (-2) \cdot (-2)(-2)2 + (-1)2 + (-2)2 = 43.$$

Answer: 43

The edges of the triangle of grade A, b, calculate the tangent length connecting the vertex of the triangle and the selected point of c edge, knowing that this point divides the edge into tangent 1 and c 2 of length c.



graph4-

amusement. Let CAB triangle CA=a, CB=b, AB=c, AL=c1, L = c 1, LB = c 2, c=c1 + 1 "searched incision" (Figure 2.3.8).

Obviously, what. Then CA= CL + LA and CB= CL + LB

CA2 = (CL + LA) 2, CA2 = (CL + LA) 2, where to come from

A 2 = l 2 + 2 CLLA + c12, and. b2

= l2 + 2 CL · LB + c22

It follows by multiplying the first of the resulting equation by c2 and the second is c2 and adding the resulting ratio

a2 c2 + b2 c1 = l2 c1 + c2 + 2 CL · c2 LA + c1 LB + c1 c2 c1 + c2 = l2c

+ c c1 c2, ( ) ( ) ( )

Because c2 LA

+ c1 LB θ expression is equal, it is equal to the sum of two vectors of the same length

l2c = a2 c2 + b2 c1 − c c1 c2

4.4.4 The idea of using geometric transformation in plane Mosaic graphic design

Knowing that a regular three, four, hexagon can realize the dense spread of the plane, then can an irregular figure realize the dense spread of the plane? Based on these three patterns, translation, rotation and axial symmetry. Principle of translation transformation in plane mosaic design: left and right, up and down; example of maple leaf design with square:

1. Draw a curve on the left and right sides of the square, and draw a curve on the top and bottom sides. The two curves form two shadows with the side length of the square respectively;

2. Move to the right along the horizontal side to the right side; cut the upper side and move down the vertical side to the lower side;

3. Draw the edge of the curve along the right and lower side curve, and cut out the basic figure of the following figure along the edge of the curve;

4. The following figure is the basic figure that can be inlaid in the plane. Add the texture of the blade, and it is the plane inlaid figure "maple leaf" designed. The mosaic pattern in the plane is shown on the far right of the following figure;

Figure 4-12 Maple leaf of plane mosaic design

4.5 Teaching design case of the idea of penetrating geometric transformation

Throughout the college geometry course, the textbook sets the algebraic geometry interspersed type, and has geometric content in each grade, the only difference is the number of geometric content. In teaching, grade seven and grade eight, due to the incompleteness of geometry knowledge, can gradually permeate the idea of geometric transformation. Ninth grade students have learned most of the university geometry knowledge, but a large part of students still hold a static view of geometry and lack a dynamic understanding of geometry. Students understanding level of geometry is still in the stage of paying attention to the nature, and they can not find new geometric conclusions according to the existing conditions. After research, it is found that the model examination questions and the middle examination questions are mostly adapted from some geometric figures in the textbook. Therefore, if the focus on the idea of geometric transformation, the best method is in the review class of grade 9. Change the traditional method of reviewing knowledge points and "brushing questions", so that students can deeply understand the relationship of geometric models through cooperative exploration, conjecture discovery, verifying the conclusion, and lay a foundation for students to understand the idea of geometry and permeate geometric transformation from the perspective of movement transformation.

Similar triangles are very important in college geometry learning. Students similar unit tests show that the average score of a full score of 150 is only over 80, not yet reaching the passing standard. Students mastery of this part of the content is uneven, and their understanding of similar models is confused. Therefore, in order to consolidate students understanding of similar models, this paper designs two teaching designs. The first focuses on establishing the connection between similar models from the perspective of geometric transformation, and the second focuses on using the geometric transformation to explore exercises.

4.5.1 Teaching Design 1: Inquiry on Similar and Common Model Relations

Inquiry teaching design of the common model relationship of similar triangles

Instructional design concept:

At present, the teaching of similar models is to summarize the basic figures. These basic figures are stored in the brain, and how to establish their connections and form a knowledge network is a consideration. The geometric thinking level 1 and level 2 of grade 9 students have been developed relatively perfect, and the thinking level of most students is in the excessive stage of level 3 to level 4. Therefore, this lesson is based on fan Hills geometric thinking theory. The core of teaching design from one of the similar model, through geometric transformation, with the help of intuitive and describe other similar model, so as to establish the connection between the basic graphics, students on the basis of the similar triangle model, found the connection between the model, make the isolated model formation model network, deepen the model of memory and understanding, and promote the development of students abstract thinking and formal reasoning ability. At the same time, cultivate students concept of geometric transformation, and understand the idea of geometric transformation, and internalize the transformation idea.

Knowledge teaching objectives:

1. Summarize and comb through the common similar triangle models.

2. Explore the internal connections of the similar triangle models.

Ideological teaching objectives:

1. Cultivate students dynamic geometric concepts.

2. Understand and form the idea of geometric transformation: "the constant thought in change".

Ability teaching objectives:

Develop students geometric inquiry ability and improve their geometric thinking ability.

Teaching process design:

Review review

Can you try to draw the similar triangle figure that you remember most?

Students try to draw similar triangles, and observe and show the similar triangles drawn by students.

Design intention: By drawing similar triangles, on the one hand, to test students ability to summarize and conclude in the process of geometry learning, on the other hand, to provide certain intuitive materials for the next exploration.

4 - 13 Triangle similarity model

Teacher: The similar triangles drawn by the students can be roughly divided into the following situations.

Teacher: The chapter of the movement of the graphics in the first volume of the seventh grade has learned the movement of some graphics. Do you still remember any graphic movements?

Student: Translation, rotation, axisymmetry

Teacher: What are the characteristics of the graphic movement? So, what is the mathematical idea contained in the above graph movement process?

Student 1: Through the above movement of the graphics and the original graphics are all equal, the shape only changed the position, the shape and size of the shape has not changed, that is, the side length and Angle of the shape have not changed.

Student 2: In the process of the above figure movement, the position of the figure has changed, but the shape and size of the figure have not changed, so the idea of "constant in change" is implied.

Teacher: You can see two similar triangles from the figure above. Are there any above variable invariants?

Student 3: There is no invariant, because the size of two similar triangles will change.

Student 4: There is an invariant. Since the shape of two similar triangles has not changed, there are still invariants, that is, the size of the Angle is still the same, and the degree of the corresponding Angle is invariant.

Design intention: Different students have different understandings of this problem, so as to cause students cognitive conflicts, and create conditions for the next step of cooperative exploration.

Teacher: Two similar triangles, although their sizes are different, but their shape has not changed, so the size of their Angle has not changed, the unvariable is the Angle. Although their sides are longer or longer or shorter, the ratio of the corresponding line segment does not change, so this ratio is not variable, calling this ratio the similarity ratio.

cooperative inquiry

Teacher: Can the above similar triangle use a similar figure to get other types of similar figure through its movement?

Student 5: No, they dont seem to matter.

Student 6: Some can, for example, through 180° of rotation, the first similar graph can become the second similar graph.

Student 7: Yes? If this is the case, then other similar figures should also be obtained from one of them, so that some connection is established between all of the similar models.

Design intention: Through the setting of problems, on the one hand, guide students to view similar problems from the perspective of graph movement, on the other hand, train students to view mathematics with the view of connection, which is conducive to the formation of a whole and system of mathematical knowledge.

Teacher: Do these similar models are related? The following students in the group to explore it! Using translation, rotation, axisymmetry, you can draw figures and try to reason.

Design intention: Through students cooperative exploration, review the geometric drawing points of graphic movement, and explore the relationship between similar models through drawing, find the internal connection between models, and experience the fun of discovery. In the group discussion, students have found connections between similar models. Lets take a look at the results of each ion. A group of inquiry results display (thought teaching goal: ur f rotational transformation)

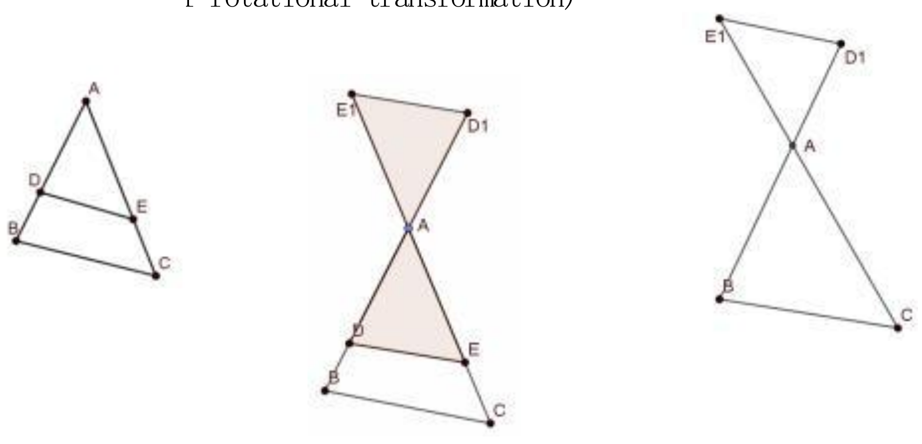


Figure 4-14 for "Type A" and "Type 8" relationships

Group representative: Starting from the "type A" similarity model, through observation and discussion, the group members found that the "type 8" similarity model can be obtained from the central symmetry of A triangle in the "type A" similarity model. Through the rotation transformation, it can be found that there is a connection between these seemingly unrelated figures.

Two groups of exploration results display

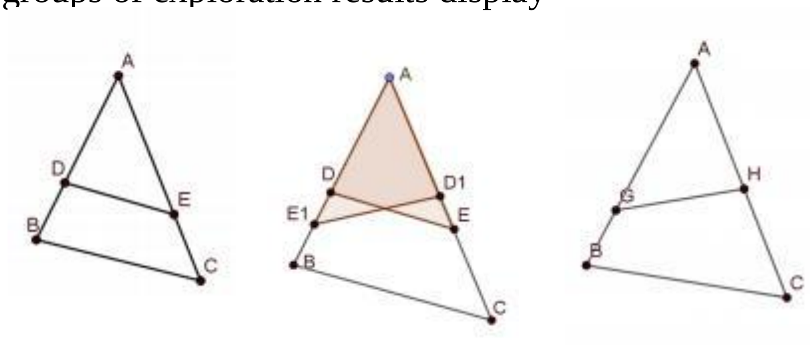


Figure 4-15 The "Type A" and "oblique Type A" relationships

Group representative: The group also started from the "type A" similarity model. Through observation and attempt to map, they found that the "oblique type A" similarity model could be transformed by the "type A" similarity model with the bisector of the Angle of the common angle as the symmetry axis.

Three g[illegible]oratic[illegible]on display

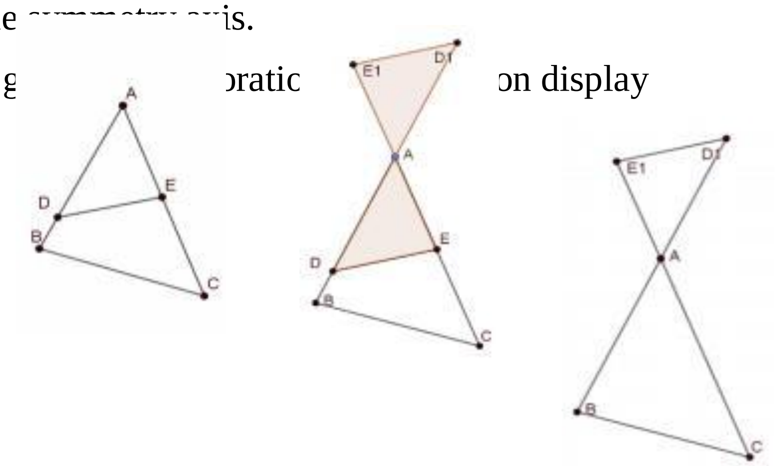


Figure 4-16 "oblique A" and "dovetail" relationships

Group representative: Starting from the "oblique A type" similar model, the group found that the "dovetail type" similar model can be obtained by rotating A triangle with A common vertex in the "oblique A type" similar model.

Four groups of inquiry results are on display

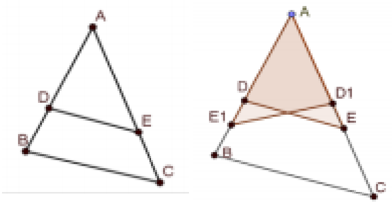
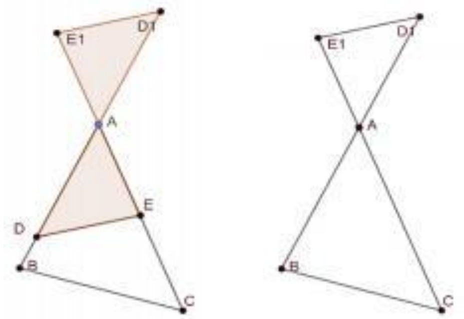


Figure 4-17 "Type A" and "dovetail" relationships



Group representative: Starting from the "type A" similarity model, some members of the group found that the "dovetail" similar model could be "oblique type A" by "type A" with the angular bisector of the public Angle as the symmetry axis, and then rotated 180° around the public vertex.

Teacher: Through our in-depth thinking and communication, each group has its own discovery, and the teacher also has their own discovery. Do the students want to listen to the teachers discovery?

Student: Think, think, teacher, let me talk about your discovery.

Figure 4-18 The dynamics of the similarity model

Teacher: if from the perspective of movement, I start from the "oblique A", then I put the line segment DE as A line, translation line DE, you can get "mother" similar model, continue to shift line, can also get "common Angle dovetail" similar model, can even by putting one of the triangle rotation to any one position, get "hand in hand" similar model.

Students, as long as you are good at thinking and exploring, there will be new discoveries. The teacher just provides the students with the idea of geometric transformation, the students can feel the role of geometric transformation, the students can continue to explore after class, if you explore a new conclusion, please share your exploration results with the students.

Summary: New findings have been made through cooperative exploration. Lets summarize the relationship between these similar models.

Design intention: Through the summary link, further consolidate students memory of common figures in similar triangles, understand the geometric transformation ideas used in them, and lay a foundation for subsequent students to understand geometry from the perspective of geometric transformation.

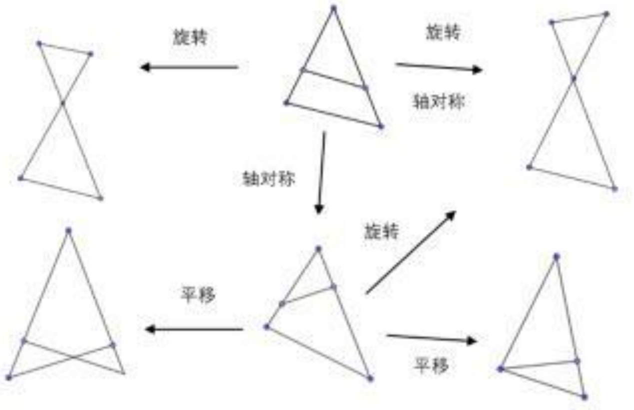


Figure 4-19 Dynamic Model dynamic dynamic relationship network

Class summary

1. What have you learned in mathematics?

2. What do you think of math learning?

to give an assignment

1. Combing the common types of similar triangles from the perspective of geometric transformation

2. Try to write down what is the condition that each model meets two triangles as similar?

3. Through these similar models, can you still make corresponding explorations and write down your inquiry conclusions.

Design intention: Through assignment 1, students are required to transfer the application of geometric transformations to a common model of understanding equal triangles. Through homework 2, break through the disadvantages of students "rote" model, so that students can understand the model in a geometric and intuitive way, and train students to demonstrate similar triangles closely. Through homework 3 (choose to do questions), it can provide an opportunity for students who are able to learn, so that students can find and put forward new questions from simple questions, and improve their high-level thinking level.

Teaching reflection

This class is a review class of similar triangles. From the perspective of geometric transformation, students are guided to focus on the development of students, explore the relationship between similar models, and design from the perspective of the development of connection. Students find the connection between knowledge through hands-on mapping and group cooperative exploration. This class not only permeates the idea of geometric transformation, but also guides students to carry out group inquiry, inquiry is not only in the mathematics class, but also in the students self-learning, students can do self-inquiry in learning, and can get considerable development in mathematics.

4.5.2 Teaching design 2: Exploration on the Thought of Permeating geometric Transformation

The teaching design of penetrating the idea of geometric transformation

Instructional design concept:

Learning is inseparable from problem solving, the traditional geometry exercises are under the known conditions, draw certain conclusions. Students on the training of the topic, the graph of the old topic and "change" into a new topic, to the students learning has brought a lot of burden. Whether we can explore the old questions from the perspective of the movement change, find new conclusions, and even promote the old questions, solve a kind of problem through the in-depth research of a topic, cultivate students concept of the movement change, exploration ability and innovation ability. Teachers should understand the development of the students thinking level, teaching to build on the original level of thinking, ninth grade students geometric thinking level most of the students van hill geometric thinking in level 3: abstract, students can not formal argument, in order to promote students thinking to level 4: the development of formal reasoning, this lesson is designed on the basis of van hill theory of. This class takes the typical topic in the textbook as an example, with the help of geometric transformation for the visualization of figure transformation, and explains geometric transformation with geometric transformation, to promote the development of students geometric demonstration ability and thinking level, so that some students geometric thinking can move to the level 4, so that students internalize the idea of geometric transformation.

Knowledge teaching objectives:

Equal triangle, similar triangles

Ideological teaching objectives:

Master, apply, and internalize the ideas of geometric transformation

Ability teaching objectives:

Based on geometric transformation, cultivate students geometric thinking.

Teaching process design:

New class introduction

In this class, explore what convenience can graph transformation bring to geometry problem solving?

cooperative inquiry

As shown in the figure, there is a point C on line segment BD, and equal sides △ ABC and CD, respectively. Line segment AD and BE intersect at point F, BE and AC intersect at point G, and AD crosses CE at point H.

Question 1: Guess the relationship between line segment BE and line segment AD and give the proof.

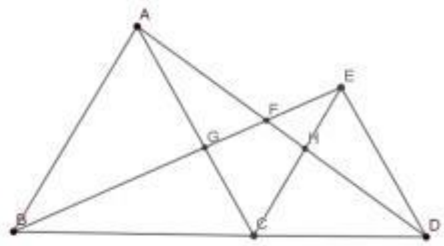


Figure 4-20 Equilateral triangle of common vertices

Teacher: Class, from your intuition, what is the relationship between BE and AD?

Student 1: BE = AD.

Teacher: Your intuition is very good. Is there any other speculation?

Student 2: Relationship refers to the quantitative relationship and position relationship. Position relationship is parallel or vertical, but as can be seen in the figure, the two lines do not seem to be vertical.

Design intention: to the students van Hill geometric thinking level 1: intuitive into the foundation, through the questions to guide the students to guess.

Teacher: This student provides a direction of thinking. The students all have their own guess in their hearts. The following students will demonstrate the relationship between the length and position of the line segment BE and AD. Students tried to prove it, and most of the students proved it.

Teacher: Which student will share the ideas of the argument?

Student 3: My idea is to get BE = AD and CAD= CBE by proving △ ACD △ BCE

BFD = BAF + ABF = BAC + CAD + ABF = BAC + ABC=120° Thus, line segment BE = AD,

And with a BFD = 120 °.

Teacher: I know that the rotation of a figure is to rotate a figure around a certain Angle in a certain direction. Before and after the rotation, the shape and size of the figure do not change. Can you see a static geometric figure from the Angle of rotation transformation?

Question 2: From the perspective of transformation, what is the motion transformation relationship between △ ACD and △ BCE? How can we understand the relationship between BE and AD from the perspective of graph transformation?

Teacher: First of all, △ ACD and △ BCE are composed of which line segments, and what is the relationship between them?

Student 4: △ ACD is composed of segment AC, CD, AD, △ BCE is composed of segment BC, CE and BE, △ ABC and △ CDE are equilateral triangles, AC=BC, CD=CE, ACC = CB = BCE = 60°; segment AC can be regarded as segment BC clockwise rotation 60° around point C, and segment CD can be regarded as segment CE C clockwise rotation 60 °.

Design intention: Through the level of students geometric thinking level 2: description, promote students new understanding of geometric argument, and promote the development of students thinking.

Teacher: The triangle is made up of line segments, but what about the rotation of the line segments?

Student 5: Oh, the △ ACD can be seen as the △ BCE around the point C clockwise rotation of 60° by the nature of the rotation of the BE=AD.

Teacher: So how to understand the BFD = 120 °?

Student 6: The line segment on the triangle also rotates for 60°, that is, BFA = 60 ° and BFD = 120 °. The tational transformation makes the problem more intuitive and conducive to solving geometry. Now let the graph move, and each group tries to make the graph and explore.

Question 3: Take C as the center of rotation, rotate △ CDE, whether the relationship between BE and AD changes during the rotation process, make a figure and briefly explain the reasons.

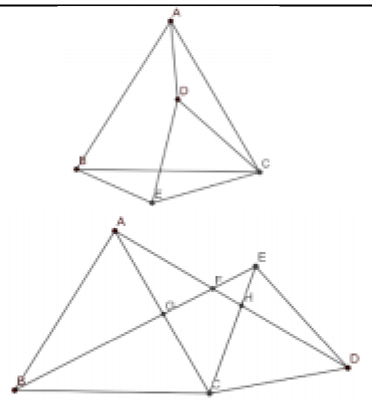
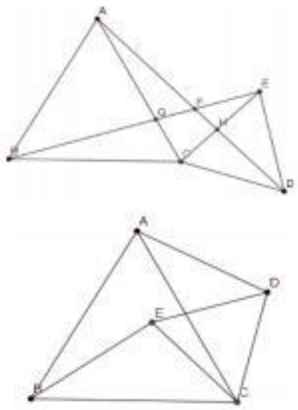


Figure Figure 4 - 21 The dynamic model

Design intention: cultivate students dynamic concept and geometric intuition, and permeate the constant thought in the process of movement transformation. As the two triangles are unchanged during rotation, the number and position relationship of line segments will not change.



Question 4: What other conclusions can you draw? Write down your conclusions and explain why.

Design intention: Cultivate students thinking ability through open questions.

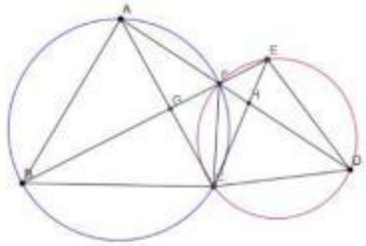


Figure 4-22 The dynamics of the similarity model

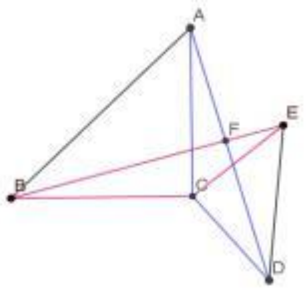
One set of conclusions: obtain similar triangle AFB = ACB = 60 ° , △ AGF △ BGC; △, △ ABG △ FCG; △ EFH △ DCH; △ CFH △ DEH.

Conclusion of the two groups: the four points of A, B, C, F and C, D, E, F are obtained, so it is easier to see the similar triangles above.

Question 5: From the perspective of graph rotation, can you make up a similar question based on this question? Draw out the figure, and write out the title you wrote.

Student compiled topic presentation:

1. As shown in the figure: in △ ABC and △ CDE, BCA = DCE = 90 °, CA=CB, CD=CE, line segment BE and line segment AD intersect at point F, verification: BE = AD, BEAD.



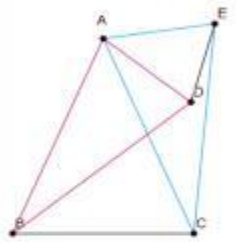


Figure 4-32 A generalization of the similarity model

2. As shown in the figure: in △ ABC and △ ADE, BAC = DAE, AB=AC, AD = AE, connecting line segment BD, AE, proof: BE = AD.

Design intention: Through the topic compilation link, change the status quo of students passive topic, so that students can compile different levels of difficulty according to their own ability, so that students can further understand and internalize the idea of geometric transformation.

summarize

I conclude a question. I think you students have a deep understanding of the idea of rotation.

Class summary

Tell me what you have learned in this lesson!

Student 7: In this class, I reviewed the proof of equal triangle and similar triangles.

Student 8: From the problem from the perspective of movement transformation, simple questions can also gain a lot.

Student 9: The rotation transformation includes invariants. Seeing the problem from the perspective of rotation, the conclusion is more intuitive.

Student 10: Dig deeper into some geometric figures, and there will be different findings.

Homework layout

Topic 1.2.3 (required questions)

Continue to explore the example questions of this lesson (optional questions)

Design intention: this lesson for the ninth grade section old topic new inquiry class, this lesson on the basis of the original cognitive structure and geometric thinking level, with the aid of geometric intuitive, through the form of set problem string, stimulate students thinking, pay attention to geometric transformation thought penetration, reflect "change in the same, in the same should change", pay attention to cultivate students inquiry consciousness and innovation ability.

Admidia V section
The teaching experiment of geometric transformation thought infiltration

This chapter mainly carries out the teaching experiment of geometric transformation thought infiltration. Because the infiltration experiment is conducted in the classroom, this paper adopts the quasi-experiment research method. The main contents include: teaching experimental objects and processes, experimental hypothesis, experimental test tools, and analysis of experimental results. At present, the evaluation methods of learning tend to be diversified, and this experiment adopts diversified methods to analyze the experimental results. Analysis of experimental results: statistical analysis of questionnaire before and after the experiment, data analysis of results before and after the experiment, case comparison of geometric test results after the experiment, analysis of sound thinking after the experiment, and teaching suggestions for the infiltration of geometric transformation ideas.

5.1 Subjects and procedures

The teaching experiment was carried out in a university in Qingpu District, Shanghai. After the statistical analysis of the class performance in the early stage of the experiment, classes 9 (2) and 9 (12) of the ninth grade were selected as the experimental subjects. Class (2) and Class (12) are parallel classes with the same math level and those led by the same math teacher. This teacher has 15 years of teaching experience and has been teaching in grade 9 in recent years, and he is familiar with the nine years of geometry teaching. Among them, Class 9 (2) is the experimental class, with the teaching intervention of geometric transformation thought infiltration; Class 9 (12) is the control class, without teaching intervention according to the original teaching method. To ensure the effectiveness of the experiment, other relevant variables such as teaching content, teaching progress, and teachers teaching methods remain unchanged.

The arrangement of school teaching schedule, the content of similar triangle is arranged in the second volume of grade eight, and the teaching of vector content and the review of similar triangle are conducted at the beginning of grade 9. Through similar triangle to before the experiment test, found that the average grade of two classes did not reach the pass level, so the teaching experiment is based on the review of similar triangle, teaching intervention experiment time for September 11,2020 to November 6,2020, twice a week for 14 classes teaching, teaching intervention time arrangement in the afternoon of the last self-study class.

In order to make geometry learning more interesting, several activity classes are designed in the experiment, so that students can learn geometry knowledge and feel the idea of geometry transformation in the activities. Experimental course content: graphic shear splicing experience geometric transformation thought activity class, the introduction of geometric transformation, geometric transformation relationship to explore special courses, geometric transformation thought under the understanding of the essence, the relationship of similar model, plane Mosaic graphic design with transformation practice class, geometric transformation under the geometric comprehensive development class, try to transform a problem problem.

5.2 Experimental hypothesis

There are five experimental variables during this experiment:

Independent variable: the teaching of geometric transformation thought infiltration is divided into two levels: the teaching of geometric transformation thought of the experimental object, the teaching of geometric transformation thought of the experimental object.

Dependent variables: students mathematical performance, geometric inquiry ability, mathematical thinking ability, and non-intellectual factors.

Control variables: teachers teaching method, students learning style, teaching content, teaching progress and other irrelevant variables.

Experimental mode: single-factor between-subject design.

Hypothesis 1: The geometry teaching with the idea of geometric transformation can improve students understanding and attention to geometric transformation.

Hypothesis 2: The geometry teaching of geometric transformation thought infiltration can improve the math performance of grade 9 students.

Hypothesis 3: The geometry teaching with the idea of geometric transformation can improve the geometric inquiry ability of grade 9 students. Students with different math scores have differences in their ability to accept geometric transformation.

Hypothesis 4: The geometric teaching of geometric transformation thought infiltration can improve the ninth grade students geometric thinking ability and the ability to solve one problem more often.

5.3 Experimental test tools

After the end of the experiment, the test tool is the student questionnaire and geometric test paper. The focus of the questionnaire survey is the students cognitive tendency in the non-intellectual factors of geometric transformation, and the focus of the test paper is the students intellectual factors, mainly including the application of geometric transformation in problem solving, and the students comprehensive ability to analyze and explore geometry.

In the past Shanghai model examination and middle school examination papers, the geometric score accounts for a high proportion, and the geometric part accounts for more than 100 points. The examination score can be used as a way to test the teaching practice. The infiltration of mathematical thinking method cultivates students thinking, further studies the teaching practice to solve problems with geometric transformation, and whether the geometric exploration ability is improved.

Posttest geometry paper, the first, two small questions are selected from a similar triangle exercise book to fill in the blanks, the original focus of the topic is parallel and similar content. The questions has not changed in this test paper, but the focus is on the application of geometric transformation and the ability to solve one question. In the study of thinking, Wang Hongbing set the open test questions on the basis of analyzing the single test situation in the existing geometric thinking level test. Although targeted, it could not reflect the broad limitations of students thinking. Students answer the open questions to present their inquiry thinking status. Therefore, on the basis of the typical questions in the textbook, this paper compiled an open question, corresponding to the third question of the post-test geometry paper, its purpose is to examine the geometric transformation concept and geometric exploration ability of students with different grades.

For students geometric test, in order to further understand the students geometry problem solving thinking process, the experimental class math scores is divided into before 33%, 33%, 34%, randomly select a student as a representative of the level, three representative samples for separate test, testing in the process of recording equipment recording thinking process, and recycle the subject to do the test volume, in the process of loud thinking tester does not give any hints.

5.4 Analysis of the experimental results

5.4.1 Statistical analysis of the student questionnaire before and after the experiment

In this study, through the distribution and collection of questionnaires and the data statistics, the students in the experimental class have their understanding of geometry learning and the importance of the idea of geometric transformation. The specific data analysis results are presented as follows:

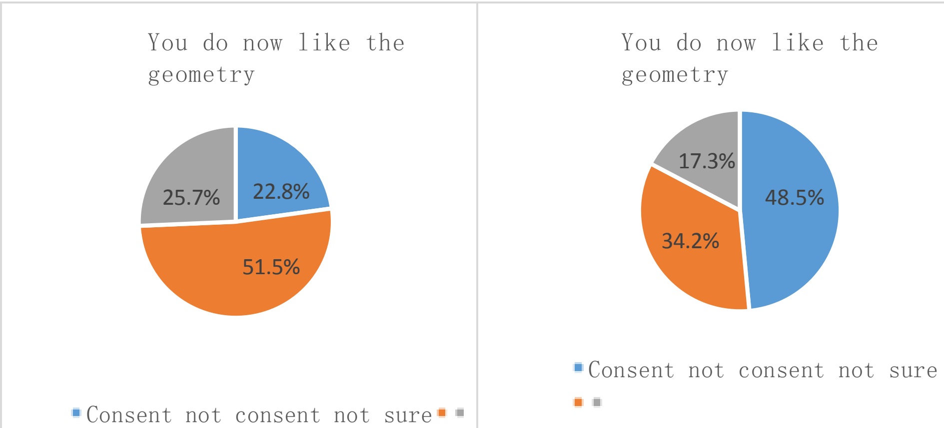


Figure 5-1 Before infiltration of experimental class Figure 5-2 After infiltration of experimental class

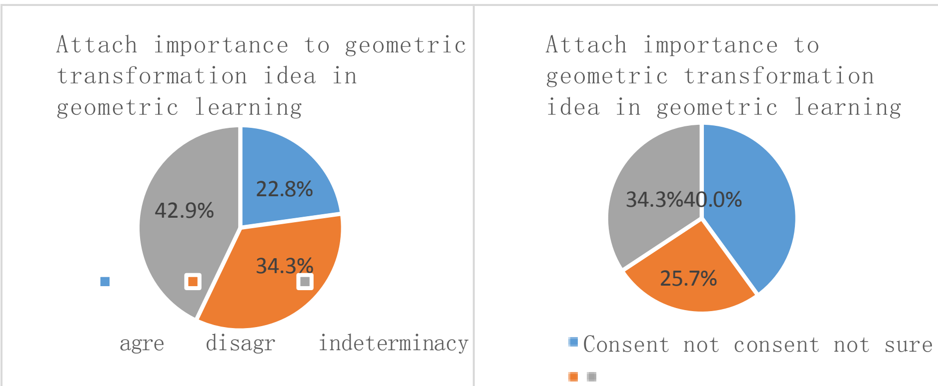


Figure 5-3 Before infiltration of experimental class Figure 5-4 After infiltration of experimental class

According to the statistical analysis results of the questionnaire data, we can see that through the teaching experiment of geometric transformation thought infiltration, students love for geometry has been significantly improved. Specifically, the percentage of students who like geometry increased from 22.8% before the experiment to 48.5% after the experiment; Students attention to the idea of geometric transformation has also been significantly improved, As the percentage of valued students increased from 22.8% before the experiment to 40.0% after the trial, The percentage of students who did not value it also decreased from 34.3% before the experiment to 25.7% after the experiment, The percentage of uncertain students also decreased from 42.9% before the experiment to 34.3% after the trial; It shows that the infiltration teaching of geometric transformation thought can change students understanding to some extent, The effects on the students non-intellectual factors are more obvious. Through the comparative analysis of the questionnaire results of the experimental class, hypothesis 1: the teaching experiment of geometric transformation thought infiltration can improve the ninth grade students understanding and attention to geometric transformation.

The questionnaire data before and after the experimental class are as follows:

Table 5-1 Questionnaire data results before and after the experiment in Class 9 (2)

		agree	disagree	indeterminacy
3. You think that geometric transformations are helpful in solving geometric problems.	around	60.0%	22.8%	17.2%
		71.4%	17.1%	11.5%
4. You often use geometric transformations to solve geometric problems.	around	28.5%	34.2%	37.3%
		62.8%	20.0%	17.2%
5. You think that geometric transformations are helpful in the learning of geometric concepts.	around	57.1%	14.3%	28.6%
		66.7%	11.4%	21.7%
6. You think that geometric transformations are helpful for the addition of helpers.	around	34.2%	37.1%	38.7%
		62.8%	11.4%	25.8%
7. You can understand the relevant ideas contained in the geometric transformations.	around	25.7%	54.2%	20.1%
		54.2%	22.8%	23.0%
8. The idea of geometric transformation deepens your understanding of geometric models.	around	34.2%	34.2%	31.6%
		54.2%	37.1%	8.7%
9. The idea of geometric transformation makes you understand the nature of auxiliary line additions.	around	28.5%	42.8%	28.7%
		51.4%	22.8%	25.8%
10. You take the initiative to analyze the graphics from the perspective of geometric transformation.	around	34.2%	22.8%	43.0%
		54.2%	14.2%	31.6%
11. You will first try to solve the problem by using a geometric transformation method.	around	28.5%	48.5%	23.0%
		45.7%	34.2%	20.1%
12. After you solve the problem, you will continue to make the geometric transformation exploration.	around	11.4%	42.8%	45.8%
		28.5%	37.1%	34.4%

Through the analysis of the questionnaire data, the students in the experimental class have greatly improved their understanding and application of geometric transformation. For example, the proportion of students who believe that geometric transformation deepens the understanding of geometric model increased from 34.2% before the experiment to 54.2% after the experiment; the proportion of students who actively solve the geometric transformation from 28.5% to 45.7% after the experiment; the effect is not obvious; the proportion of students increased from 11.4% before the experiment, indicating that penetration teaching in this aspect needs to be further strengthened and further studied in detail.

5.4.2 Data analysis of mathematics academic achievement before and after the experiment

New semester begins, the school held the school test, a week after the similar triangle test, the experiment began with similar triangle after unit test, before and after the experiment and nine (12) class five math test: school test, similar unit, month, midterm, final results, with SPSS25 independent sample t test, processing results are as follows:

Table 5-2 Independent sample t-test of the two classes before and after the experiment

number	Item	average value	standard deviation	t price	P price
1	Opening examination (2) class Class examination (12)	94.34 93.80	22.98 20.68	0.199	0.843
2	Similar unit (Class 2) Similar unit: (Class 12)	89.51 87.49	28.14 29.83	1.395	0.172
3	Monthly examination (2) class Monthly examination (12) class	100.40 97.66	26.86 25.03	0.588	0.560

4	Mid-term (2) class	106.06	24.43	2.557	0.015*
	Midterm (12) class	98.29	25.10		
5	Final (2) class	113.91	22.17	3.685	0.001
	Final (12) class	102.46	25.23		

According to the data analysis, the P-value of the independent sample t-test for the mean scores of the unit test scores of similar triangles were 0.843> 0.05 and 0.172> 0.05, indicating that there was no significant difference in the math scores of the first two classes in the 95% confidence interval, laying the foundation for the validity of the experiment.

During the experiment, the P-value of the independent sample t-test for the monthly test scores of the two classes was 0.56> 0.05, indicating that there was no significant difference between the monthly test math scores in the 95% confidence interval, and the experimental effect was still not significant in the short term. During the experiment, the P-value of the independent sample t-test was 0.015 <0.05; the P-value of the independent sample t-test for the final exam results was 0.001 <0.05, indicating that the math scores of the midterm and final exams of the two classes were significantly different in 95% confidence interval, and the average score of the experimental class was significantly higher than the average score of the control class. To some extent, it shows that the teaching experiment of geometric transformation thought infiltration has not achieved a significant effect in improving the average score of students in the short term, but it has a significant effect in improving the average score of students. The statistical analysis of math results verifies hypothesis 2 to some extent.

Further study the difference in the teaching effect of the infiltration experiment on students with different grades, and calculate the progress rate of the experimental class. The calculation method of the progress rate is the number of progress divided by the total number of the class, and the progress standard: the average score of the midterm and final grades is greater than that of the opening examination and the marking score of similar units. The average of the first two scores of the experiment was calculated according to the high, medium and low groups, and the progress rate is shown in Table 5-3.

Table 5-3 Statistics of progress rate before and after the experiment

	Low grouping	Midgroup	High group	amount to
experimental class	58. 3%	75.0%	63.6%	65.7%
Control class	50.0%	50.0%	45.4%	48.5%
Group number	12	12	11	35

Table 5-4 Chi-square test for progress rate

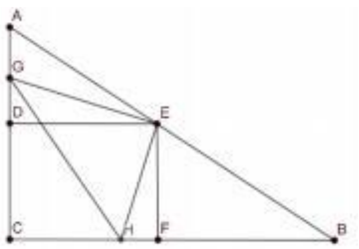
	Class (average score ± SD)		χ^2	P
	Class 2	Class 12		
Progress rate	66%±0.08	48%±0.03	8.0	0.238

The statistics show that the progress rate of each group in experimental class is 58.3%, 75.0%, 63.6%, and the progress rate of class is 65.7%; the progress rate of each group in control class is 50.0%, 50.0%, 45.4%, and the progress rate of class is 48.5%. The results of the chi-square test for the progress rate in the two classes showed that P=0.238> 0.05, indicating no significant difference in the progress rate between the two classes. The difference of group performance from low progress rate is not obvious, the medium group and high group are about 20%; and the medium group is better than the high group; the teaching experiment for students at different levels, in which the significant degree of teaching effect is medium group> high group> low group. To some extent, it shows that medium grouping is a very potential group in the teaching practice of geometric transformation thought infiltration, and the teaching effect of low grouping is not obvious.

An in-depth analysis of the reasons behind this result is made in theory. Theoretically, the students in higher groups make more progress. In fact, the higher group than the higher students, which also explains the reason why the progress rate of the students in higher groups is higher than that of the higher group. Because of their weak basic knowledge of low group students, the teaching effect of low group is not significant. In order to study the influence of penetration experiment on students geometric thinking, because the thinking process belongs to the implicit psychological activities, so facilitate the research need to thinking process explicit, using the method of vocal thinking, requires students to say the thinking process, the whole process of recording, sorting and analysis of audio materials.

△ The AGE E rotates counterclockwise around the point E Turn to 90° for a △ MEH; After the point G

C, H, E, G, and the tangent value of △ CED △ GEH GHE is unchanged rotates, the corresponding point is



∠GED=∠HEF
△GED≌△HEF

△ GED around point E 90° counterclockwi

The conditional CDEF is known to be a square, with AE = 5 and BE = 7

From the rotation properties, point A rotates to get the corresponding point on

If a square is changed to a rectangle, what are the

△ GED and △ HEF incomplete so the area and change, incomplete will still be similar?

5.4.3 Thinking analysis of geometric test

Question 1. In the figure, the four vertices are on the edge of △ ABC, AE = 5, EB=7, G and H are the moving points on the edge of AC and BC, and whether GEGE, S △ AGE + S △ BEH are a fixed value? If the result is found for a fixed value, if it is not a fixed value, please explain the reason.

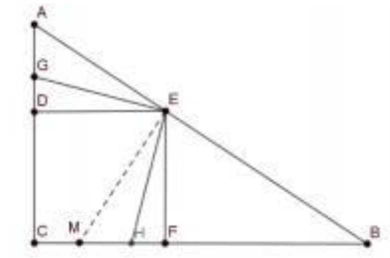


Figure 5-5 square Figure 5-6 rectangle

The audio data of the sound thinking of the students experiments are compiled into the following thinking flow chart:

Conclusion can be concluded: DEEF, EDDC, EFCFDE = EF = CD = CF; GEHE

GE=EH, ∠EGD=∠EHF
∠AGE+∠EGD=180
°

S△AGE+S△BEH=S△BEM
S△BEM=×5×7=$\frac{135}{2\ 2}$
SCGEH invariant, GE=EH

△GED∽△HEF
GEDE
=
EHEF
△AED∽△BEF

Figure 5-7 Flow chart of sound thinking (Test 1)

The flow chart of the students sound thinking shows that the students solution is different from the calculation method of the unknown number, parallel line and Pythagorean theorem used in the pretest results. The first thought of the student is to get the corresponding segment equality and triangle equality according to the known conditions, the student thinks of the view of motion transformation, including rotation transformation. The area of the two triangles required by the question is transformed into the area of a triangle. Then it is to find the sum of the area of the triangle. The student once again uses the properties of rotation to obtain the triangle of the required area is a right triangle. Driven by the idea of rotation transformation, the original complex calculation problem is solved quickly and easily. The student continued to think that the area and unchanged of the two triangles are equivalent to the area of the remaining quadrilateral, which is a conclusion beyond the topic the student got. The student continues to think, if the square in the question becomes a rectangle, then is the area sum of the two triangles a fixed one? What are the other invariants? As shown in Figure 5-6, compared with the above practice, it is found that the triangle is no longer equal, and the area and also accordingly, but there are still invariants in the process of transformation: the shape of △ GEH remains unchanged.

Using the thinking of students thinking process visualization, through the analysis of students thinking flow chart, the following conclusion: students can think and analyze the problem from the perspective of geometric transformation, thus improve the concept of the students, the students can general thinking, if the square into a rectangle what invariants? In this process, the invariant in the process of transformation is also found, which to some extent explains that the teaching practice of the infiltration of geometric transformation thought expands the depth and breadth of students thinking, and verifies hypothesis 4 to some extent.

Test question 2: As shown in the figure, in the square grid with a side length of 1, find ABC + ACD =.

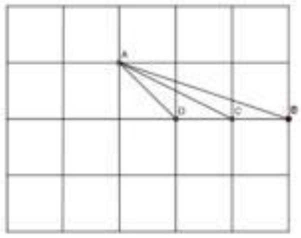


Figure 5-8 for lattice triangles

The test question 2 in the post-experiment test paper aims to test students tendency to use geometric transformation and their ability to solve the next question of geometric transformation, so as to reflect the level of students divergent thinking.

Judging from the sound thinking flow chart of test question 2, the first step is to complete the answer with a similar triangle. After solving, he used the idea of geometric transformation and made three figures corresponding to three solutions by using geometric transformation. The method of Figure 5-9 is to make an axisymmetric transformation of a line segment to transform the position of the angles, convert the two separate angles into one angle, and finally obtain the sum of the two angles by solving the triangle. For the method shown in figure 5-10, a line segment two geometric transformation, first using an axisymmetric transformation to realize the position of the Angle, but did not achieve the desired effect, again transform the line segment to realize the position of the Angle, the two Angle into a Angle, finally by solve the triangle of the two angles. In FIGS. 5-11, each line segment is transformed separately, one line segment is axisymmetric to realize the position transformation of one angle, and the other line segment is transformed to transform the position of the other angle. Finally, the two angles are concentrated in the third position, and the sum of the two angles is obtained by solving the triangle.

Although these three solutions belong to the same kind, these three solutions all reflect the improvement of students thinking breadth, the teaching experiment of infiltrating the thought of geometric transformation, plays a role in promoting the improvement of students thinking breadth, and verifies hypothesis 4.

According to the conditions, we can find that the line segment BD=2, CD=1, AD= √2, AC= √5, AB= √10

Request two corners and, to find a way to convert two

① $\frac{AD}{DC}=\frac{BD}{AD}$

∠ADC=∠ADB

△ ADC∽△ ADB

∠ABD=∠CAD

∠ABC+∠ACD =∠CAD+∠ACD=45°

② Two corners are converted to one angle using

The AC is transformed axisymmetrically along the AE

AC1, ACD = CAE = C 1 AE ABC= BAE linked as

Change the AC axisymmetrically along the CD to AC1, ACD = A1 CD, along the CB direction

Translation A1C is one

In △ ABC1, AC1=BC1= 5AB= 10, AC1 BC1 was obtained by the Pythagorean theorem, C1AB=45°

∠ABC+∠ACD=45°

In △ ABC1, AA2=BA2= 5AB= 10, AA2 by Pythagorean theorem BA2, A2BA=45°

∠ABC+∠ACD=45°

AC is transformed axisymmetrically transformed to A1 A2, A2 A1C= ACD, A1 B1 ABC= BA1 B2, connecting A2 B1, that is, A2 A1 B1= ABC + ACD similarly

∠ABC+∠ACD=45°

Figure 5-9 Flow chart of sound thinking (Test 2)

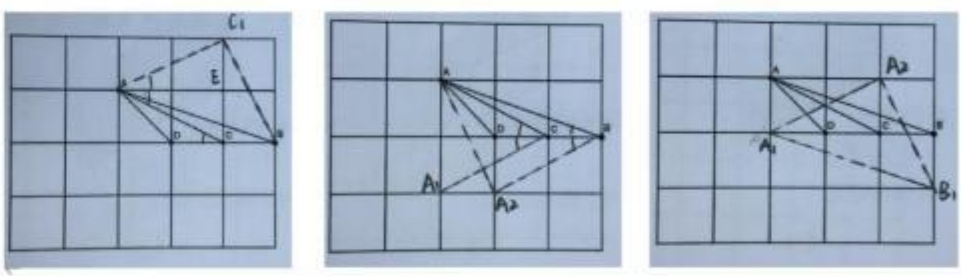


Figure 5-10 AC Axisymmetric Figure 5-11 AC first axis symmetry after translation Figure 5-12 AC Axisymmetric AB translation

So what is the effect of the teaching of penetrating geometric transformation thought in cultivating students ability of geometric inquiry? In order to study the influence of teaching experiment on students using geometric transformation to solve geometric problems, a targeted geometric test paper was compiled. A student in the low, middle and high groups was randomly selected in the experimental class to conduct geometric test, and the test results were compared and analyzed in individual cases.

5.4.4 Case comparative analysis of geometric test results

This question corresponds to the third question of the post-test geometry paper, and the prototype of the first question is derived from a class exercise of the eighth grade triangle of the last teaching edition. The first question is the basic proof of the basic knowledge of mathematics; the second question is to understand the equivalent triangle and the setting is the effect of examining the idea of geometric transformation. The third question is the examination of the similar triangle on the basis of geometric transformation ideas. Geometry test results for low-group students:

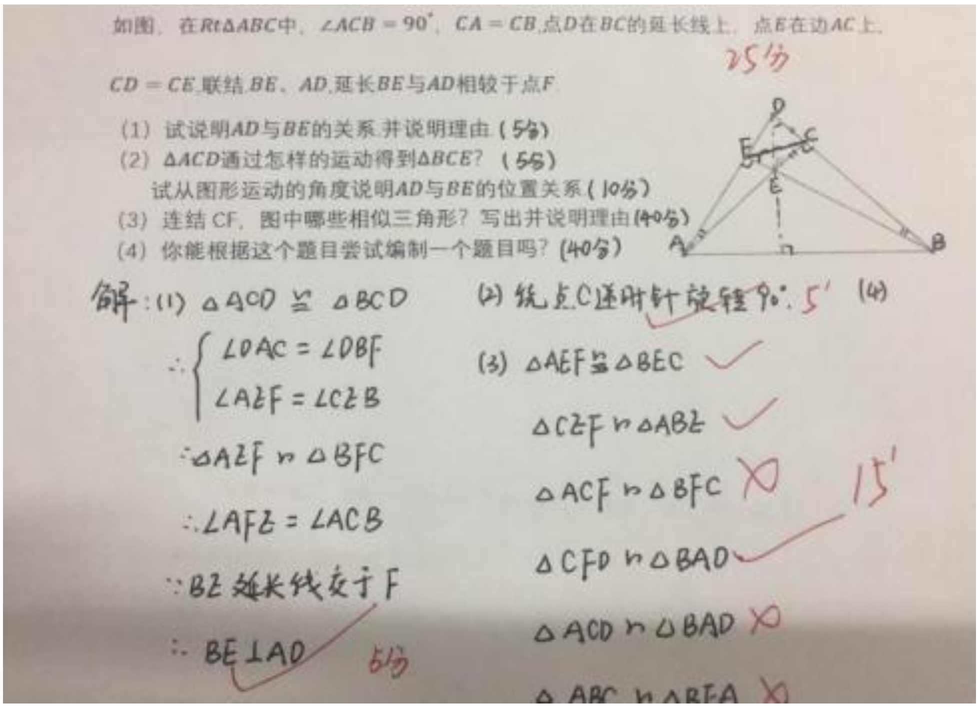


Figure 5-13 Test results of low-group students

It can be seen from the test results that the low-group students have a good grasp of the judgment of equal triangles, but they are not optimistic about the knowledge of similar triangles. The graphics in the topic include many basic models of similar triangles, and this student only wrote three pairs, and did not reason according to the requirements of the topic. To understand the complete triangle from the perspective of geometric transformation, it can only identify the basic geometric rotation, but can not explain the position relationship of line segments from the perspective of geometric transformation. The similar results written by the students show that the student has a thinking set, and the similar basic figures in the teaching process are only stuck at the level of memory, do not understand the conditions satisfied by the basic figures, and do not thoroughly understand the essence of the rotation, and the ability of inquiry and innovation is insufficient. In short, the infiltration effect of geometric transformation idea on students with low groups is not very obvious.

There are several reasons why the penetration effect of geometric transformation on students with low groups is not obvious: students with low groups belong to the group with relatively weak foundation. Because of these students "fear of difficulties" in learning geometric transformation, the hands-on activity class of penetrating geometric transformation can be used in the teaching. On the premise of cultivating interest, through the graphic cutting activities, students can not only consolidate the foundation, but also feel the idea of geometric transformation. In teaching, we can adopt the infiltration of basic geometry concepts and theorems to gradually improve the ideas of geometry transformation of students with weak foundation, so that geometry teaching can have certain teaching value. Geometry test results for secondary order students:

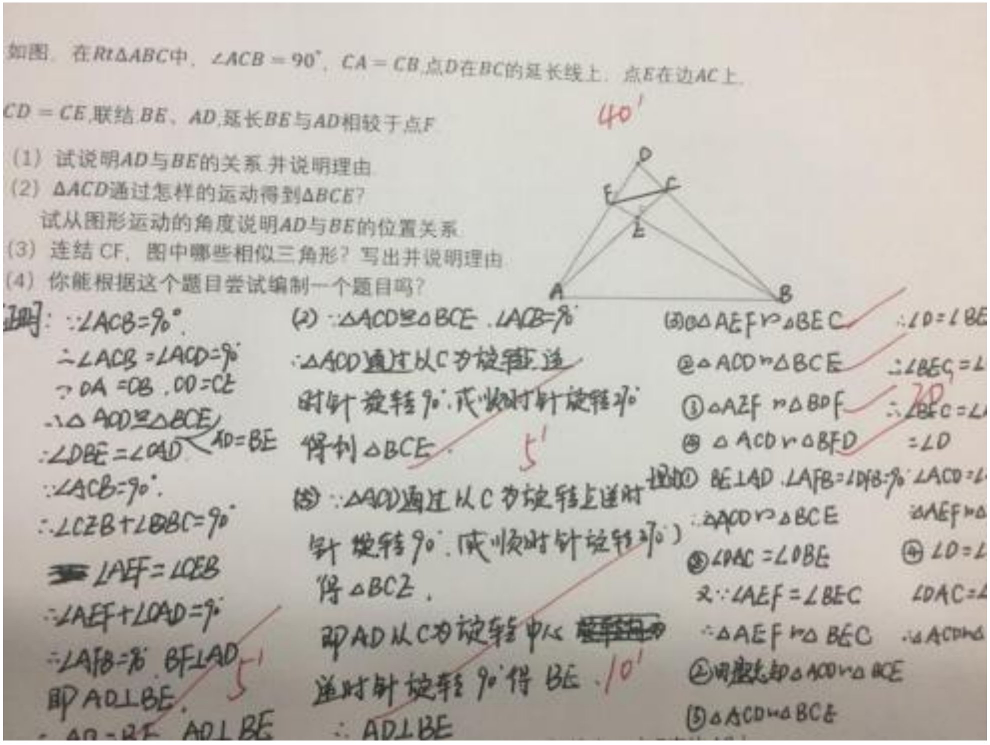


Figure 5-14 Test results of middle-group students

It can be seen from the test results of students in middle groups that students in middle groups have a good grasp of the basic knowledge of equal triangles, and the relevant knowledge of similar triangles needs to be further strengthened. This student, according to the requirements of the topic, made a certain proof of theory. From the level of thinking, he had a comprehensive and in-depth understanding of geometric transformation and could understand the basic property that any line segment corresponding to the graph rotation of 90° rotates 90°. It shows that the geometry teaching of the thought infiltration of geometric transformation has achieved a certain teaching effect among the middle group students, but the student has not reached the stage of creative thinking, so it is necessary to further penetrate and guide the thought of geometric transformation.

Geometry test results show that the penetration transformation of geometry teaching to promote students understanding of the nature of geometric problems, from the perspective of geometric transformation to understand the position relationship between AD and BE is more intuitive than geometric argument, although the current exam to use the geometric transformation method of acceptance is not clear, but can provide students with a dynamic perspective of thinking, to improve the problem solving ability.

Geometry test results of high group students:

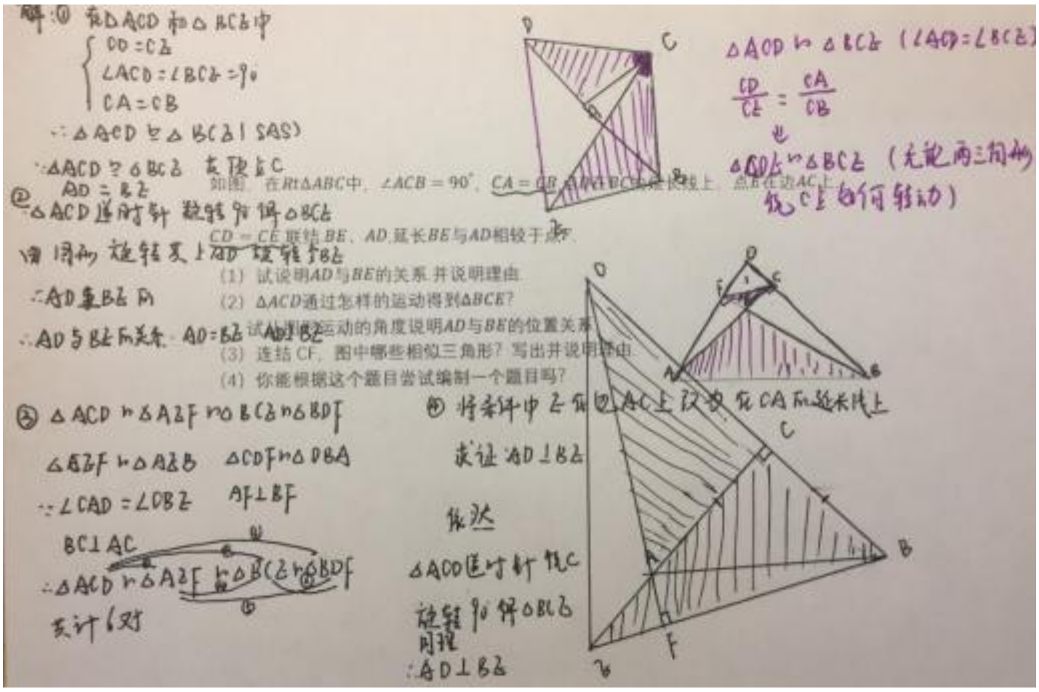


Figure 5-15 Test results of high-group students

Can be seen from the test results of high group students, the students of equal and similar the basic knowledge of more solid, the understanding of geometric transformation thought is profound, to understand the nature of the geometric transformation "constant" change, the student has strong innovation ability, able to promote the topic and reach the level of preparation questions, which can be seen that the geometric transformation thought penetration, the role of mathematics for students thinking development is more obvious, to some extent illustrates the penetration geometry transformation thought geometry teaching is fruitful.

To sum up, from the perspective of the exploration of geometric topics, students with different mathematical scores in the experimental class have different acceptance degree of the idea of geometric transformation, and the acceptance ability of the idea of geometric transformation, students with different grades have different performance, specifically as follows: high group students> middle group students> low group students. The results of the case comparative analysis verify hypothesis 3 to a certain extent: students with different math scores have differences in the acceptance of geometric transformation. In terms of the understanding of geometric transformation, high group students> middle group> low group, among which high group has the most in-depth understanding. For outstanding students of geometric transformation thought, can improve the students ability to analyze problems and expand the ability to explore, can even further penetrate the idea of geometric transformation group, can also transform into the function, the geometric transformation thought in quadratic function, proportional function, a function, etc.

5.5 Teaching suggestions for the infiltration of geometric transformation ideas

1. In daily lesson preparation, teachers should have a sense of research, study teaching materials deeply, and excavate the infiltration knowledge, activities and exercises of geometric transformation ideas. In the teaching process, teachers should dare to break through the shackles of conventional standard answers, and pay attention to guide students to learn and think from the perspective of geometric transformation. Break through the ceiling of cognition, advocate students to explore geometry, encourage students to solve one problem more.

2, the infiltration of the idea of geometric transformation should be stratified according to the actual level of students. For students with difficulty in geometry learning difficulties, we should focus on the infiltration of geometry transformation in the teaching of basic knowledge. For example, dynamic geometric figures can be used to attract students interest in learning, consolidate the basic knowledge through the movement of graphics and geometric transformation, and on the premise of students solid basic knowledge, and then carry out the infiltration of geometric transformation ideas in geometric problem solving.

3. For students above medium level, students should pay more attention to geometric transformation, seek the internal connection between knowledge through geometric transformation, and appropriately guide students to understand geometric figures from the perspective of geometric transformation, use geometric transformation for the variation of geometric questions, and grasp the invariants in the transformation process.

4. For some excellent students, students should be guided to develop to a deeper level. We can train students to develop the development of geometric exploration ability and comprehensive ability under the concept of geometric transformation, and even guide students to try to compile test questions, which can make students ability to a higher level.

Adventitia section

Conclusions and Outlook

Study Conclusion

1. Analysis of the current situation of the infiltration and application of geometric transformation ideas in university geometry teaching

Through the questionnaire survey of teachers, it is found that teachers have a positive attitude towards infiltrating the idea of geometric transformation in geometry teaching. However, in the actual teaching practice, restricted by various teaching factors, the infiltration of this thought is not satisfactory. According to the results of the questionnaire survey, students have a positive attitude towards geometric transformation, and they recognize the positive role of geometric transformation thought on geometry learning. However, due to the failure of teachers to effectively penetrate this idea in daily teaching, most students have a weak consciousness when using geometric transformation to solve problems. The further test results show that most students do not use the geometric transformation method when solving geometric problems, and their ability to explore geometry and solve the multiple of one problem under the concept of geometric transformation are at a low level.

2. An effective strategy for infiltrating the idea of geometric transformation in geometry teaching in grade nine

In the process of college geometry teaching, it is crucial for teachers to improve their own geometry literacy. Teachers should set up a high-level view of geometry teaching, especially pay attention to modern mathematical thought, and fully realize the teaching value of geometric transformation. On the one hand, teachers can refer to relevant academic research results or independently conduct research on geometric transformation; on the other hand, they should study the teaching materials, carefully organize the content carriers that can penetrate the geometric transformation ideas, and systematically integrate the teaching. Specifically, can guide the students through the graphic cutting activity feeling geometric transformation thoughts, in the process of exploring the graphic transformation relationship to deepen the understanding of geometric transformation thought, with the help of exploring a problem solution way to help students master the geometric transformation thought, use of plane graphic design and problem exploration activities, let the students to flexibly use geometric transformation ideas. In the whole teaching process of infiltrating the idea of geometric transformation, we should closely combine the actual situation of students, pay attention to every link of students from feeling and understanding to internalize the idea of geometric transformation, so that students can truly realize the powerful function of geometric transformation.

3. Analysis of the promoting effect of the teaching of the geometric transformation thought on the geometric learning of grade 9 students

The teaching of the idea of geometric transformation can change students static concept of geometric learning and promote them to establish the geometric learning view of motion transformation. After receiving this geometry teaching with the idea of penetrating geometric transformation, students will pay more attention to the application of geometric transformation thought when learning geometric knowledge. They can try to understand the geometric model from the perspective of geometric transformation, construct the geometric knowledge system, and apply it to the geometric problem solving. Students can also carry out geometric exploration from the perspective of geometric transformation and try to solve one problem. This series of changes make students thinking level significantly improved.

According to the statistical analysis of the results of the experimental class and the control class, there is no significant difference between the experimental class and the control class in the results of the monthly math examination. However, there were significant differences between the scores of the mid-term and final examinations, and the performance of the experimental class was significantly higher than that of the control class. This shows that in the short term, the teaching of the idea of geometric transformation is not effective to improve the average score of the class and improve the average score of the class. In terms of the cultivation of students thinking ability is concerned, geometric transformation can improve the thinking level of students above the middle level to a certain extent.

4. Discussion on the effect of geometric transformation in promoting students of different degrees

For different degrees of students, there are obvious differences in these promoting effects. Students with excellent grades already have strong ability to understand and apply knowledge. After contact with geometric transformation ideas, they can quickly integrate them into their existing knowledge system, and flexibly use them in the solution of complex geometric problems and extended exploration. These students can often independently find more ideas and methods based on geometric transformation, through geometric transformation to further deepen the understanding of geometric nature, expand the depth and breadth of thinking, the thinking level and problem solving ability to further sublimation, especially in the face of comprehensive, innovative geometry advantage is more outstanding.

Students with middle grades have made a great change in their learning attitude and methods after receiving the teaching of geometric transformation ideas. They began to look at geometric problems from a new perspective, and gradually mastered the skills of using geometric transformations to simplify the problems, and improved the efficiency and accuracy of solving problems. Geometric transformation provides them with a way to break through the learning bottleneck, helps them to build a more complete geometric knowledge network, enhance the comprehensive application ability of geometric knowledge, so as to gradually improve their performance, exercise the flexibility and logic of thinking, and change from a single understanding of geometric knowledge to multiple applications.

For students with poor grades, although they can also feel the changes brought about by the idea of geometric transformation to a certain extent, they will encounter more difficulties in understanding and using geometric transformation to solve problems due to the weak basic knowledge and learning habits. However, this kind of teaching method still can stimulate their interest in the learning geometry, guide them to try to think geometry from different angles, in the solution of simple geometry using geometry transformation consciousness, gradually lay a foundation for subsequent learning, gradually improve their understanding of geometry knowledge and master degree, narrow the gap with other students in geometry learning.

The idea of geometric transformation promotes the geometric learning of students of different degrees, but there are differences in the degree and form of the promotion effect due to the different foundation and ability of students. Teachers should pay full attention to this difference in the teaching process and teach them in accordance with their aptitude, so that every student can achieve the maximum development in the infiltration teaching of geometric transformation thought.

Insufficient studies

Through the study of the infiltration and application of geometric transformation ideas in university geometry teaching, the relevant research conclusions are drawn. Influenced by the practice time and other factors, this study has the following deficiencies: the geometry teaching experiment of geometric transformation thought infiltration was only conducted for two months, and the experiment time was short. The experiment is limited to two classes in grade 9, and the experiment scope is not wide enough to only reflect the teaching effect to a certain extent. Whether it is feasible to promote it needs to be tested by further expanding the experimental samples, which is limited to some extent. The influence of the influence of the thought of geometric transformation on the development of students thinking is not deep enough, and it is one-sidedness to reflect students thinking status only through the test results and "loud thinking".

As a kind of mathematical thought, geometric transformation enriches the teaching practice study of university mathematics thought and provides a new perspective for the study of university geometry teaching. Through the continuous exploration of geometric questions from the perspective of geometric transformation, the "sea of questions" mode in the traditional geometric teaching is changed, and the students inquiry consciousness and thinking ability are cultivated and exercised.

Outlook and thinking

(1) Outlook

The shortcomings of this paper are also the direction that the author needs to carry out in-depth research in the future. In view of the problem of short teaching experiment time, the author will carry out long-term infiltration teaching practice in the future teaching. This paper only teaches the idea of geometric transformation, which can be taught from the beginning of university geometry learning. For the analysis of the experimental results, the follow-up study will develop a more detailed evaluation program to track the students thinking process in more detail.

In this paper, the idea of geometric transformation can only be permeated in geometry teaching, and the idea of geometric transformation can be infiltrated into algebra, function, as well as high school trigonometric function and conic curve. It is hoped that in the future, the examination questions can set up the exploration questions infiltrating the thought of geometric transformation, so as to test the students innovation ability and application ability, and constantly emphasize the idea of geometric transformation in the examination, and even allow the language of geometric transformation in the problem solving of the examination.

(2) Thinking

Through the current situation analysis and empirical research of geometric transformation ideas in university geometry teaching, it can be found that although teachers and students hold positive attitudes towards geometric transformation, they still face many challenges in practical teaching. Teachers need to improve their geometric literacy, and effectively penetrate the idea of geometric transformation through diversified teaching methods such as graph cutting, multiple solutions for one problem, etc. Research shows that this teaching method can significantly improve students geometric exploration ability and problem-solving skills, especially in the medium-and long-term effect. Students with excellent grades can quickly apply the idea of geometric transformation to complex problems, students with middle grades can improve the efficiency and accuracy of solving problems, while students with poor grades can gradually cultivate the consciousness of geometric transformation through the practice of simple problems. Therefore, the infiltration teaching of geometric transformation thought has a positive role in promoting students at different levels. Teachers should pay attention to individual differences and teach students in accordance with their aptitude, so that every student can make progress in geometry learning.

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